Approximately Linear-Phase Recursive Digital Filters with Variable Magnitude Characteristics

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Abstract

A filter structure based on the parallel connection of a delay and a variable fractional delay all-pass filter is proposed for implementing complementary low-pass/high-pass approximately linear-phase recursive filters with variable magnitude characteristics.

The filter optimization is performed in two basic steps:
1) An initial filter is generated using a simple design scheme.
2) This filter is used as a start-up solution for further optimization being carried out by a nonlinear optimization algorithm.

The performance and the complexity of the proposed filters are compared with those of other variable digital filters proposed in the literature. This comparison shows that the number of multipliers for the proposed filters is less than 30 percent compared with other existing structures.

Proposed Filter Structure

The transfer functions for the variable digital filter pair are given by

$$H_{0,1}(z,\ \mu) = \frac{1}{2}\left[z^{-(N-1)} \pm A(z,\ \mu)\right].$$

Here, $A(z,\ \mu)$ is an Nth-order variable fractional delay all-pass filter as expressible as

$$A(z,\ \mu) = \frac{z^{-N}C(z^{-1},\ \mu)}{C(z,\ \mu)},$$

where

$$C(z,\ \mu) = 1 + \sum_{n=1}^{N} a_n(\mu)z^{-n} = 1 + \sum_{n=1}^{N}\left[\sum_{p=0}^{P} c_{pn}\mu^p\right]z^{-n}.$$

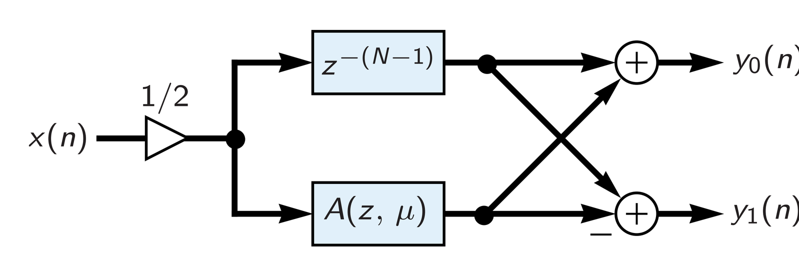

Fig: Proposed structure.

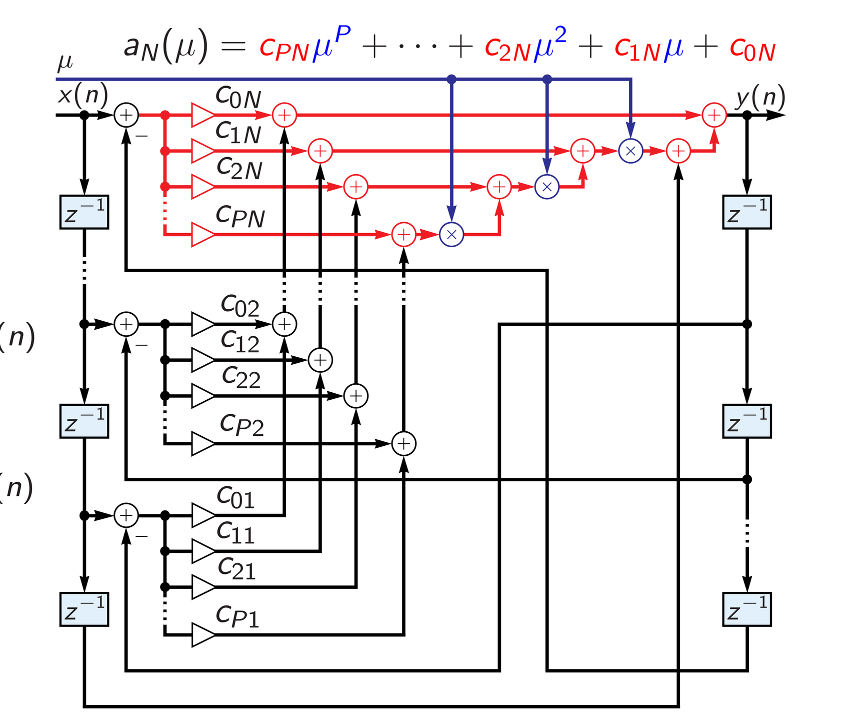

Fig: Fractional delay all-pass structure $A(z,\ \mu)$.

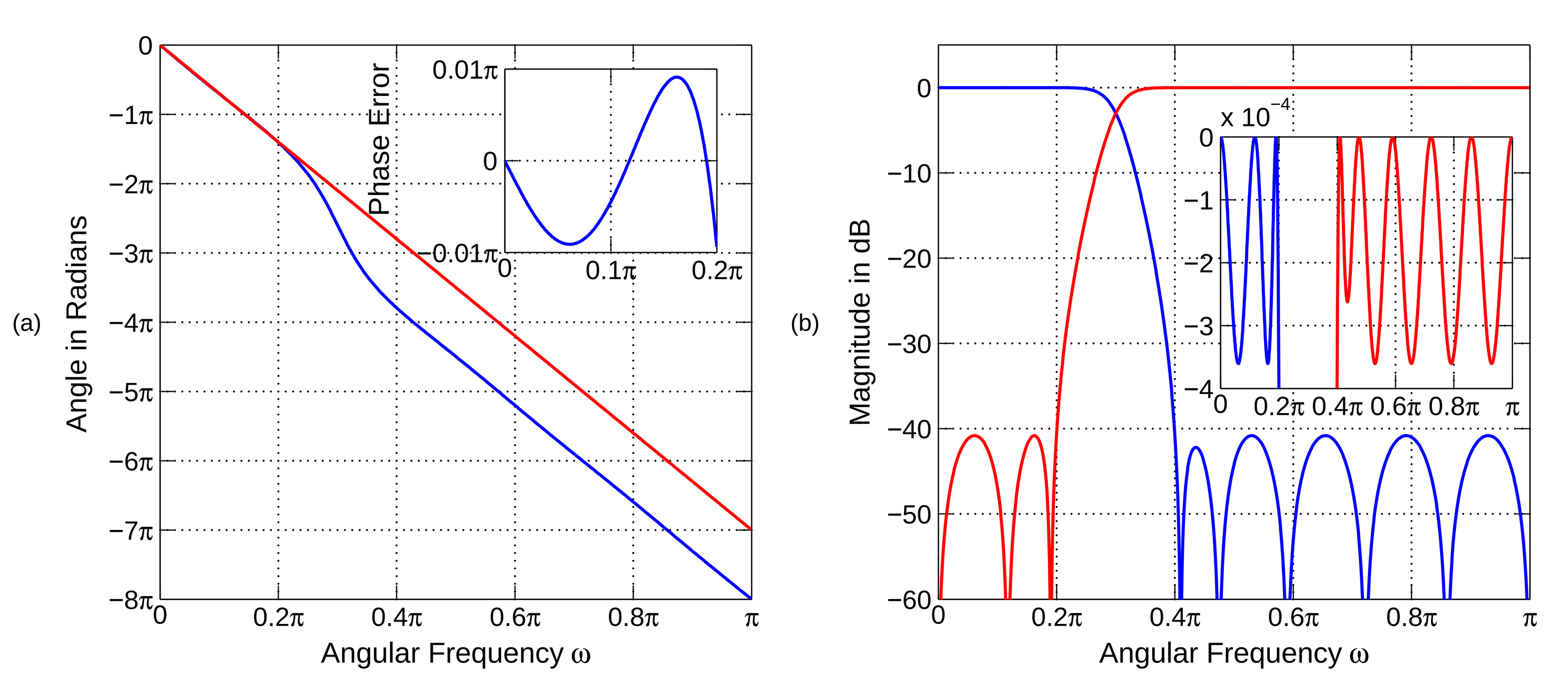

Fig: Optimized complementary low-pass/high-pass filter pair for $\mu = -1$ in Example 1. (a) Phase responses of $z^{-(N-1)}$ and $A(z,\ \mu)$. (b) Magnitude responses of $H_0(z,\ \mu)$ and $H_1(z,\ \mu)$.

Properties

The transfer functions $H_0(z,\ \mu)$ and $H_1(z,\ \mu)$ form a doubly complementary pair, satisfying both the all-pass complementary and the power complementary properties, that is,

$$H_0(z,\ \mu) + H_1(z,\ \mu) = z^{-(N-1)}$$

$$|H_0(e^{j\omega},\ \mu)|^2 + |H_1(e^{j\omega},\ \mu)|^2 = 1.$$

The doubly complementary property guarantees that where one filter has a passband, the second one has a stopband and vice versa.

For practical stopband attenuation (at least 30 dB), the passband ripple becomes very small. Therefore, the optimization of the overall complementary low-pass/high-pass filter pair can concentrate only on the stopband regions of the filters.

Optimization Goal

Determine the adjustable parameters such that for each value of μ within $0 \le \mu \le 1$, the magnitude responses of $H_0(z,\mu)$ [$H_1(z,\mu)$] closely approximates unity [zero] for $\omega \in [0,\ \omega_p + \alpha\mu]$ and zero [unity] for $\omega \in [\omega_s + \alpha\mu,\ \pi]$.

Statement of the Problem

Given the filter order N, the polynomial degree P, the passband and stopband edges, and α, **find** the adjustable parameter vector Φ to minimize the maximum absolute value of the magnitude errors as given by

$$\epsilon = \max\{\epsilon_0(\Phi),\ \epsilon_1(\Phi)\}, \quad \text{where} \quad \epsilon_k(\Phi) = \max_{-1\le\mu\le 1}\left[\max_{\omega\in\Omega_{s,k}} |H_k(\Phi,\ e^{j\omega},\ \mu)|\right].$$

Here, the stopband regions of $H_0(z,\ \mu)$ and $H_1(z,\ \mu)$ are $\Omega_{s,0} = [\omega_s + \alpha\mu,\ \pi]$ and $\Omega_{s,1} = [0,\ \omega_p + \alpha\mu]$, respectively.

Filter Optimization

Optimization Algorithm: Discretize the range $-1 \le \mu \le 1$ into the points $\mu_j \in [-1, 1]$ for $j = 1, 2, \ldots, J$ and the stopband regions into the frequency points $\omega_{i,0} \in [\omega_s + \alpha\mu_j,\ \pi]$ and $\omega_{i,1} \in [0,\ \omega_p + \alpha\mu_j]$ for $i = 1, 2, \ldots, I$ and $j = 1, 2, \ldots, J$.

The resulting discrete problem is to find the adjustable parameter vector Φ to minimize

$$\epsilon = \max\{\epsilon_0(\Phi),\ \epsilon_1(\Phi)\}, \quad \text{where} \quad \epsilon_k(\Phi) = \max_{1\le i\le I, 1\le j\le J} |H_k(\Phi,\ e^{j\omega_{i,k}},\ \mu_j)|.$$

The above problem can be solved using the function `fminimax` from the optimization toolbox provided by MathWorks, Inc.

Algorithm for Finding an Initial Filter: Design a set of J optimal fixed-coefficient filters corresponding to the above parameters μ_j for $j = 1, 2, \ldots, J$. Based on the resulting coefficient sets, derive a polynomial approximation for the coefficients.

Numerical Examples

Example 1: $\omega_p = 0.30\pi + \alpha\mu$, where $\alpha = 0.10\pi$, $\omega_s = \omega_p + 0.20\pi$, and $\delta_s \approx 0.01$
Example 2: $\omega_p = 0.26\pi + \alpha\mu$, where $\alpha = 0.16\pi$, $\omega_s = \omega_p + 0.24\pi$, and $\delta_s \approx 0.07$

	Structure	A_p (dB)	A_s (dB)	Δ_p (rad)	r_{max}	N_m
Example 1	Reference	$6.2 \cdot 10^{-2}$	38.3	–	–	117
	Proposed	$4.2 \cdot 10^{-4}$	40.4	$9.8 \cdot 10^{-3}$	0.87	32
Example 2	Reference	$7.2 \cdot 10^{-2}$	23.7	$1.6 \cdot 10^{-1}$	0.83	45
	Proposed	$1.0 \cdot 10^{-2}$	26.5	$4.8 \cdot 10^{-2}$	0.81	12

Δ_p is the maximum phase error
r_{max} is the radius of the outermost pole
N_m is the number of multipliers

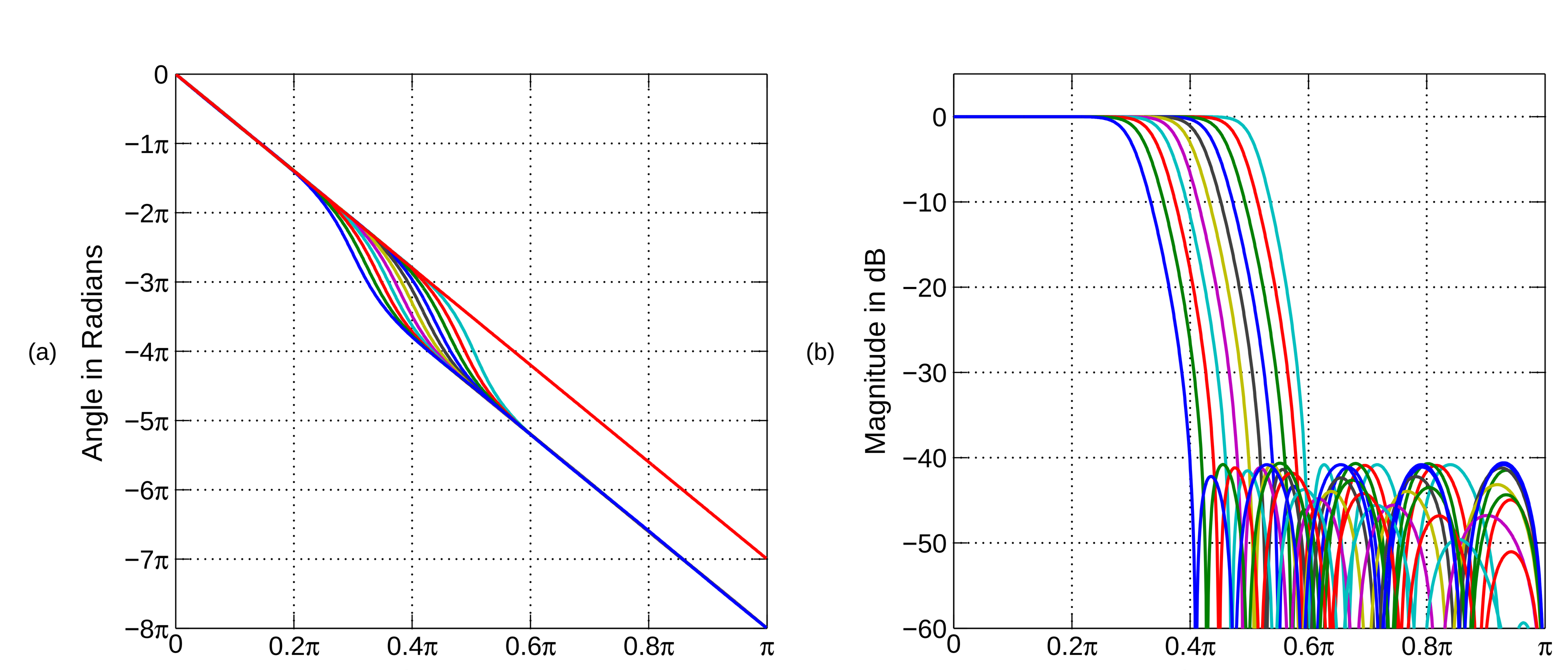

Fig: Responses for the optimized filter in Example 1 for some values of μ. (a) Phase responses. (b) Magnitude responses.