Optimization of Flexible Filter Banks based on Fast-Convolution

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Abstract

Multirate filter banks can be realized efficiently using fast-convolution (FC) processing.
The main advantage of the FC filter banks is their increased flexibility, that is,
1) the number of channels,
2) their bandwidths, and
3) the center frequencies can be independently selected.

First, a subband representation of the fast-convolution filter bank is derived.

Second, optimization problems are formulated using this model.

Third, these optimization problems are solved with the aid of the nonlinear optimization algorithm.

It is shown that the performance (selectivity) of the FC filter banks is determined by the overlap in samples in overlap-save processing instead of transform sizes.

However, the computational complexity (multiplications per output sample) reduces as the transform sizes increase.

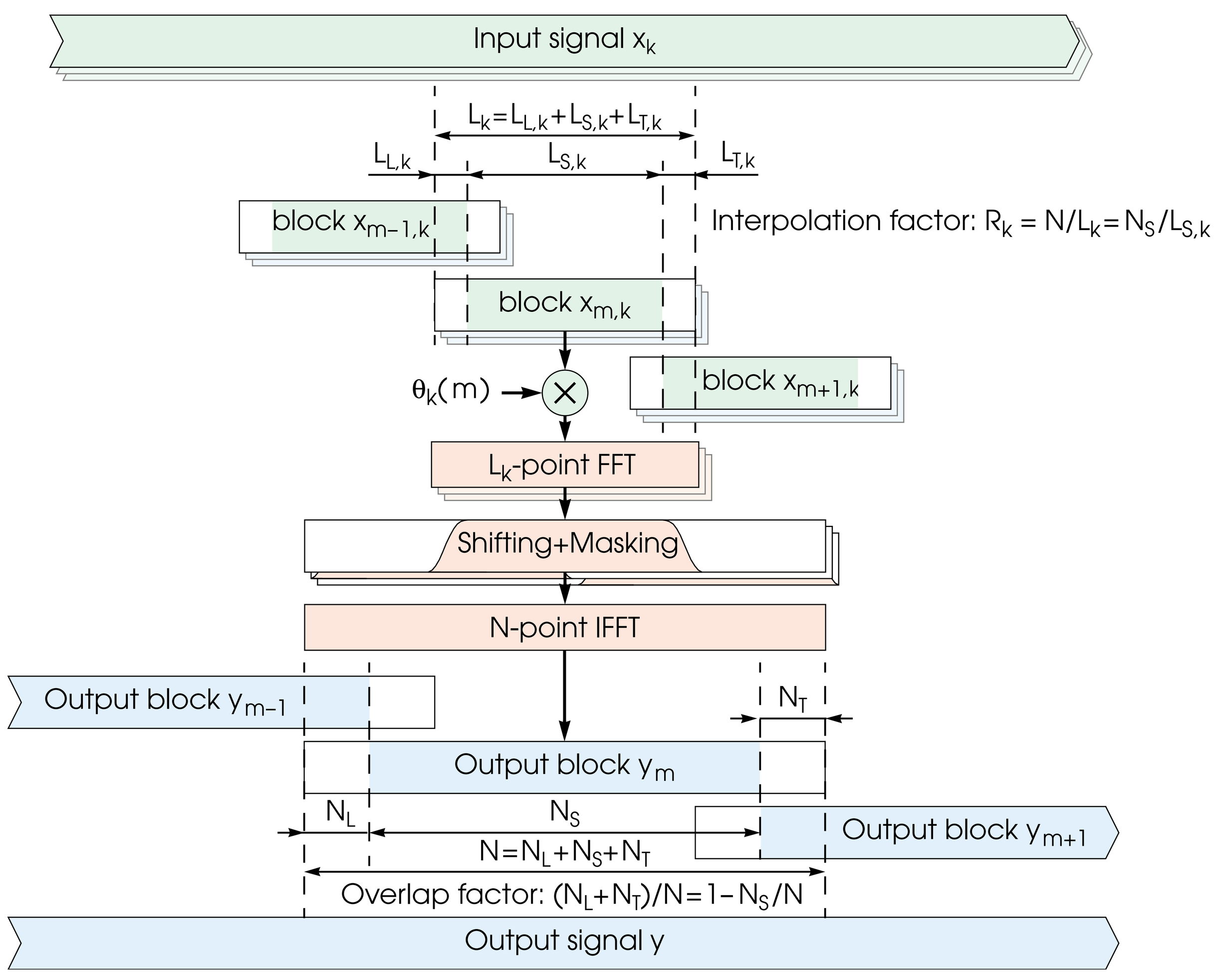


Fig: Fast-convolution filter bank with overlap-save processing.

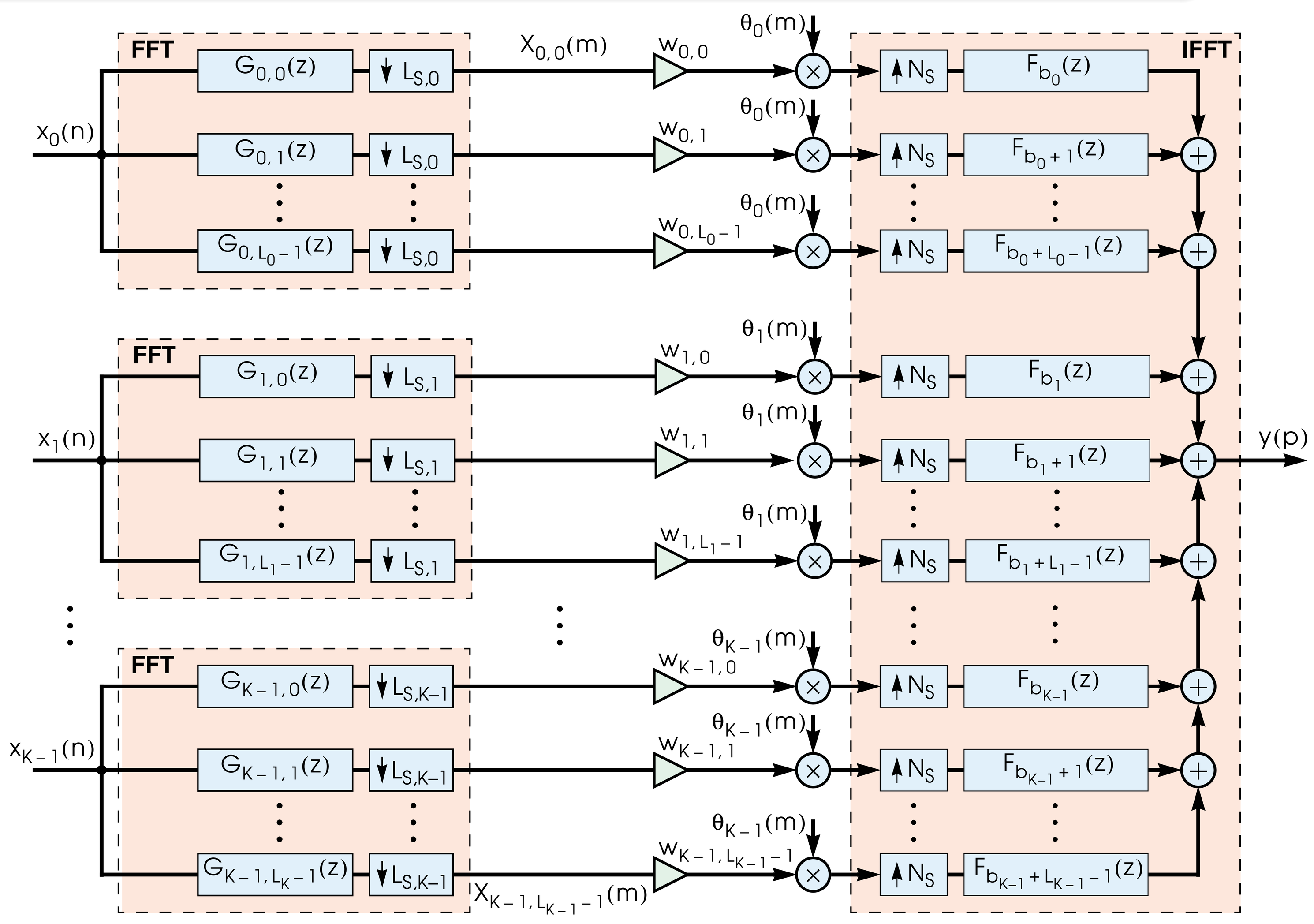


Fig: Subband representation of the fast-convolution filter bank.

Fast-Convolution Synthesis Filter Bank

Here, ***K* incoming low-rate narrowband signals** $x_k(n)$ for $k = 0, 1, \ldots, K-1$ with adjustable frequency responses and with possibly different sampling rates **are to be combined into single wideband signal** $y(p)$.

1. The incoming signals are first transformed to frequency domain using short-time Fourier transform.
2. The resulting frequency-domain signals are then shifted in frequency domain to their desired positions and weighted such that their spectra do not undesirably overlap.
3. Finally, the shifted, weighted, and combined signals are converted back to time-domain using inverse short-time Fourier transform and concatenated using overlap-save processing.

Optimization of Frequency-Domain Weights $w_{k,\ell}$

The frequency response $H_k(e^{j\omega}, \Delta)$ from the kth input to the filter output can be adjusted by the frequency-domain weights $w_{k,\ell}$.

The goal is to minimize the following function:

$$\epsilon = \max_{\substack{\omega \in [0,\omega_p] \cup [\omega_s, \pi] \\ 0 \le \Delta \le L_{L,k}-1}} |G(\omega)[|H_k(e^{j\omega}, \Delta)| - D(\omega)]|,$$

where the desired function and weighting function, respectively, are given by

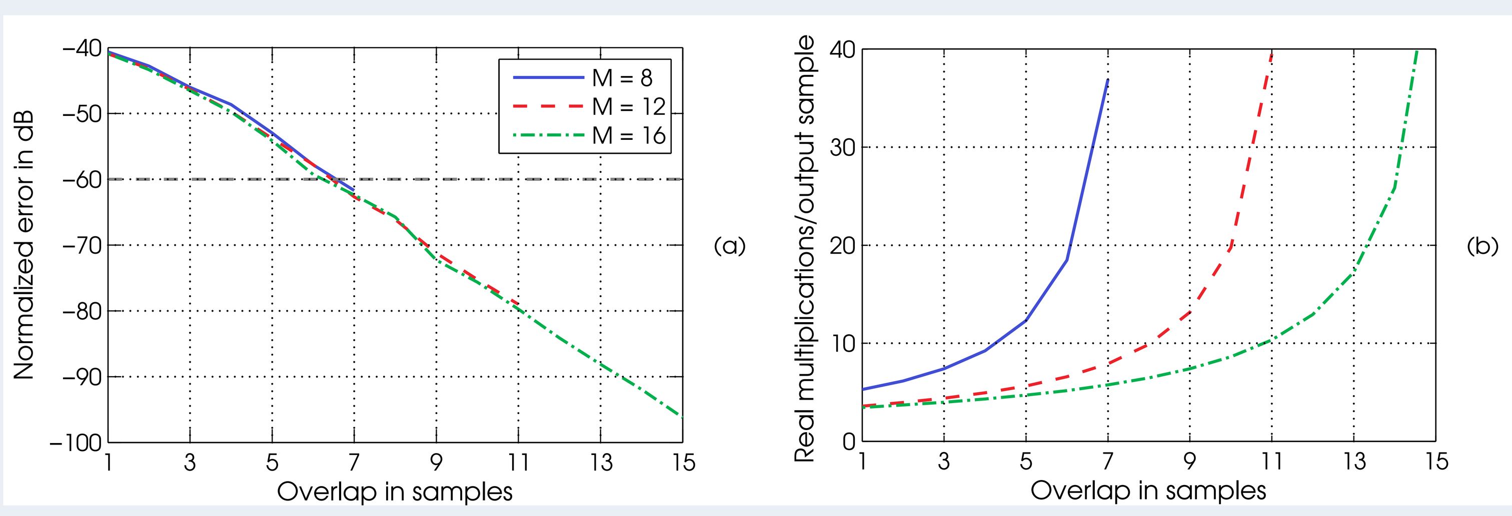
$$D(\omega) = \begin{cases} 1, & \omega \in [0, \omega_{p,k}] \\ 0, & \omega \in [\omega_{s,k}, \pi] \end{cases} \quad \text{and} \quad G(\omega) = \begin{cases} \delta_s/\delta_p, & \omega \in [0, \omega_{p,k}] \\ 1, & \omega \in [\omega_{s,k}, \pi]. \end{cases}$$

The above problem can be solved, e.g., using `fminimax` from the optimization toolbox provided by MathWorks, Inc.

Numerical Examples

Example 1: One channel ($K = 1$). Sampling rate conversion factor $R = 28/3$ with $\delta_s = 100\delta_p = 0.001$ (60-dB attenuation in the stopband).

Optimized designs: FFT size is $3M$ and IFFT size is $28M$ with $M = 8, 12,$ or 16:



Applications: Transmultiplexer in Communications Signal Processing

- FC filter bank can be used for simultaneously processing multiple communication signals of different waveform types, bandwidths, and other characteristic.
- It is possible to combine the (typically) root raised cosine-type (RRC) transmitter or receiver pulse shaping filtering with the channelization function.

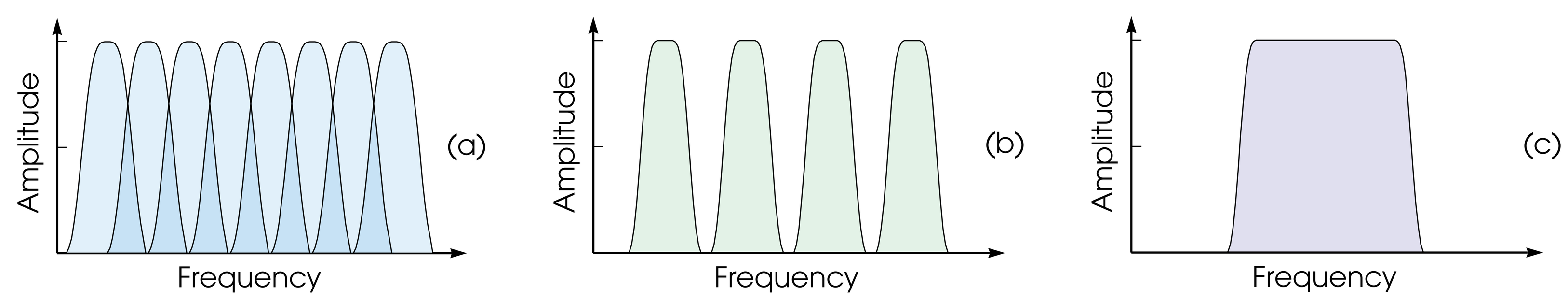


Fig: Waveforms. (a) Offset-QAM, (b) filtered multi-tone, and (c) single carrier.

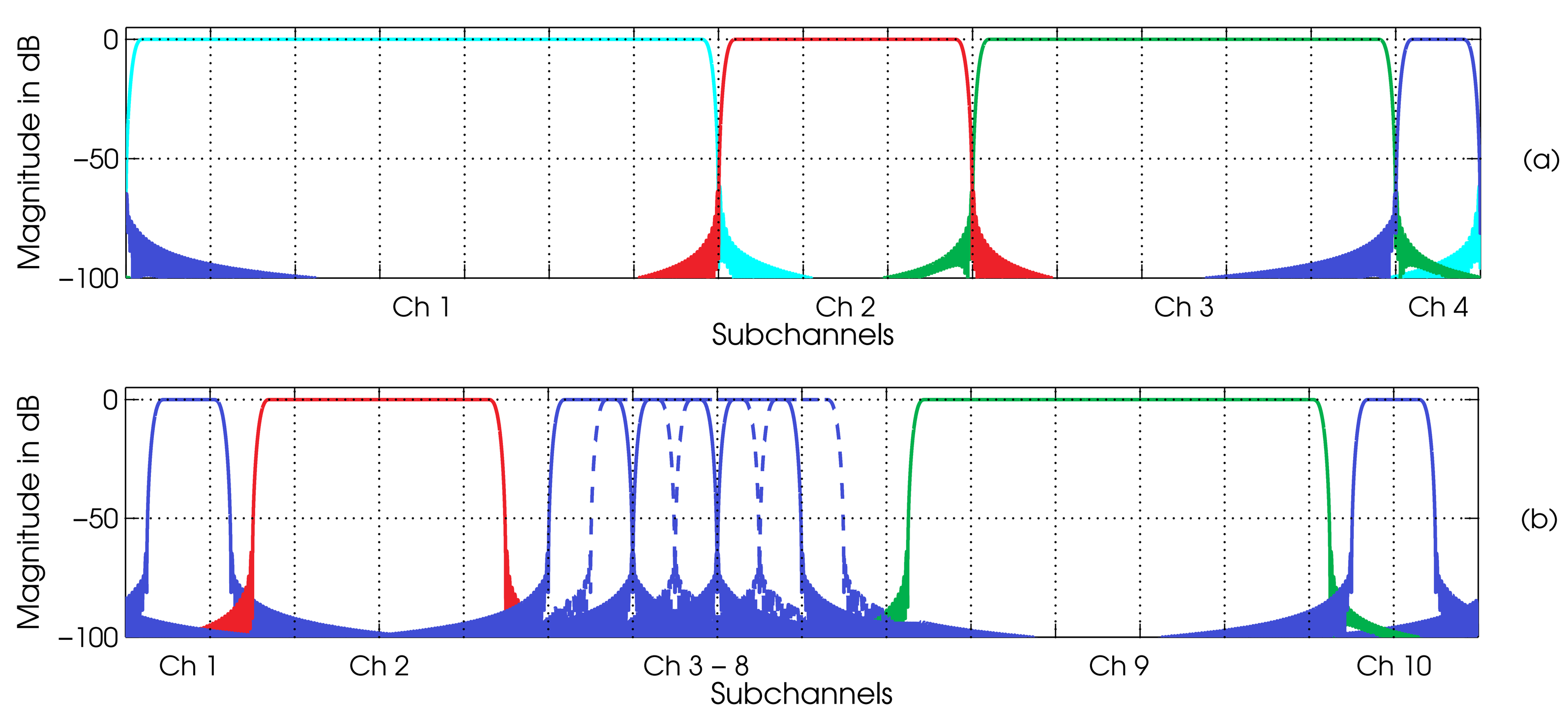


Fig: Magnitude responses for the optimized FC filter bank in Example 2.