AN EFFICIENT STRUCTURE FOR FIR FILTERS WITH AN ADJUSTABLE FRACTIONAL DELAY

Yli-Kaakinen J., Saramäki T.
Institute of Signal Processing, Tampere University of Technology
P. O. Box 553, FI-33101 Tampere, Finland
email: {ylikaaki, ts}@cs.tut.fi

Abstract. This paper introduces an efficient filter structure for implementing finite-impulse response (FIR) filters with an adjustable fractional delay. This structure is constructed by properly changing the modified Farrow structure that has been proposed by Vesma and Saramäki for the same purpose. The first subfilters are the same, whereas the remaining ones are generated using proper weighted sums of these subfilters and two very simple filters consisting of two pure delay terms. For significantly reducing the number of multipliers, including those ones required to form the above-mentioned weighted sums, the three-step synthesis scheme proposed by Yli-Kaakinen and Saramäki the case of the modified Farrow structure is followed. First, the number of subfilters and their (common odd) order is determined such that the given criteria are sufficiently exceeded. Second, an initial filter is determined using a simple design scheme. This filter serves as a start-up solution for further optimization being performed using a constrained nonlinear optimization algorithm. Third, those coefficient values of the subfilters having a negligible effect on the overall system performance are fixed to be zero-valued. In addition, some experimentally observed attractive connections between the coefficient values of the subfilters are exploited in order to reduce both the implementation cost and the parameters to be optimized even more. Both the performance and complexity of the proposed adjustable digital filters are compared with those of some existing adjustable FIR filters proposed in the literature. This comparison shows that, in the case of stringent amplitude and phase delay specifications, the number of adders and subtracters for the proposed filters is less than 80 percent when compared with the corresponding optimized modified Farrow structure.

1. Introduction

In various digital signal processing applications, there is a need for a delay that is a fraction of the sampling interval. Furthermore, it is often desired that the delay value is adjustable or variable during the computation. These applications include, among others, sampling rate conversion, echo cancellation, phased array antenna systems, time delay estimation, timing adjustment in all-digital receivers, modeling of musical instruments, and speech coding and synthesis [1, 2].

Adjustable fractional-delay filters can be designed either using finite-impulse response (FIR) or infinite-impulse response (IIR) filters. One computationally efficient technique, belonging to the former filter class, is to use the Farrow structure [1] consisting of several parallel fixed FIR filters. Recently, the modified Farrow structure has been introduced by Vesma and Saramäki in [3, 4] by properly modifying the original structure. The modified structure consists of a given number of fixed linear-phase FIR filters of the same odd order and the impulse-response coefficients alternatively possess an even and odd symmetry such that the first filter has a symmetrical impulse response. The desired fractional delay of the value μ is achieved by multiplying the outputs of these filters with quantities directly depending on this value of μ [1, 2, 4–6]. Another attractive class of adjustable fractional delay filters, belonging to the second class, are adjustable fractional delay all-pass filters based on the use of the so-called gathering structure [2, 7–9] proposed by Makundi, Laakso, and Välimäki. In this structure, the filter coefficients are the polynomials of the fractional delay parameter.

The purpose of this paper is twofold. First, an efficient structure is proposed for implementing FIR filters with an adjustable fractional delay. Second, the synthesis scheme proposed by Yli-Kaakinen and Saramäki for significantly reducing the computational complexity of the modified Farrow structure is applied to the proposed structure. This structure is constructed by making proper changes in the modified Farrow structure. In the proposed structure, the first branch filters are the same, whereas the rest of them are generated with aid of these branch filters and two simple filters by adding their outputs multiplied by constants. As in the case of the modified Farrow structure, the computational complexity of the proposed structure can be significantly reduced by using the above-mentioned synthesis scheme.

2. Proposed Structure with an Adjustable Fractional Delay

This section introduces the proposed structure for implementing FIR filters with an adjustable fractional delay. In addition, the magnitude and phase delay responses of this structure are given for the later use.

2.1. Overall Filter Structure

The proposed structure consisting of P parallel FIR filters for generating an adjustable fractional delay μ is depicted in Fig. 1. The transfer functions of the first $P-2$ FIR filters are of the form

$$G_p(z) = \sum_{n=0}^{M-1} g_p(n) z^{-n} + \sum_{n=M}^{2M-1} g_p(2M-1-n) z^{-n} \quad \text{and} \quad G_p(z) = \sum_{n=0}^{M-1} g_p(n) z^{-n} - \sum_{n=M}^{2M-1} g_p(2M-1-n) z^{-n} \tag{1a}$$

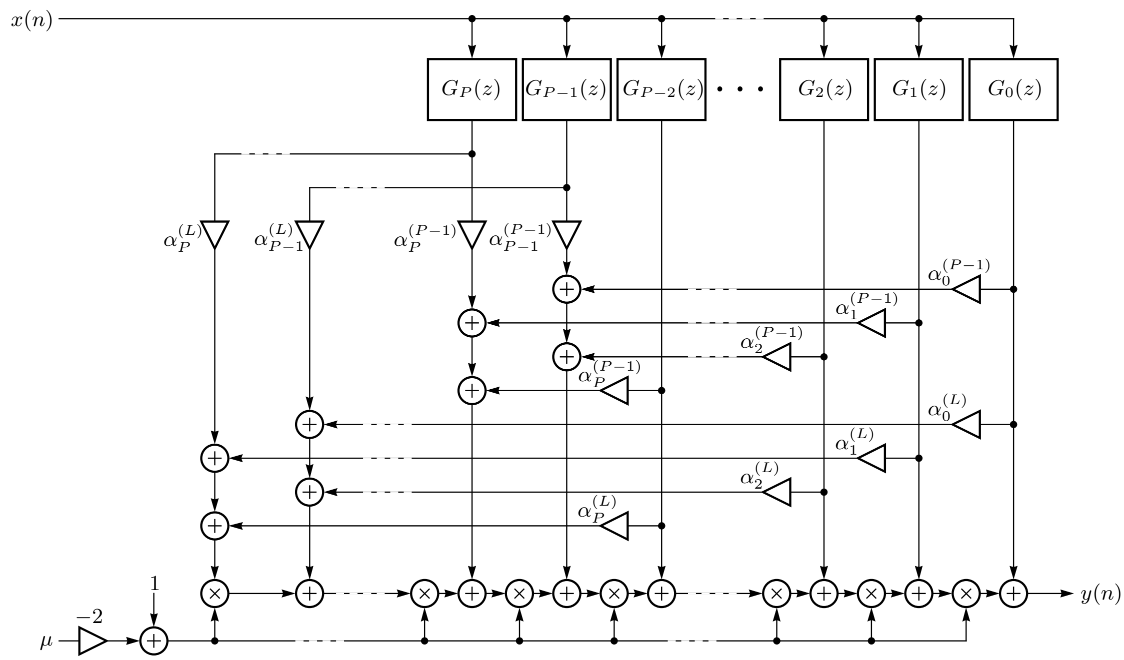

Fig. 1 A new structure with an adjustable fractional delay μ. The impulse-response coefficients of $G_p(z)$'s alternatively possess an even and odd symmetry such that the first filter has a symmetrical impulse response.

for p odd and for p even, respectively, whereas the transfer functions of the remaining two FIR filters are given by

$$G_p(z) = z^{-(M-1)} + z^{-M} \quad \text{and} \quad G_p(z) = z^{-(M-1)} - z^{-M} \tag{1b}$$

for p odd and for p even, respectively. Here, M is an integer and thus the orders of all the filters are odd.

After optimizing the impulse-response coefficients of the above subfilters as well as the additional coefficients $\alpha_p^{(l)}$ for $p = 0, 1, \ldots, P$ and for $l = P-1, P, \ldots, L$, as shown in Fig. 1, in the manner to be described later on, the role of the adjustable parameter μ in Fig. 1 is to generate the delay approximating $M - 1 + \mu$ in the given passband region. This parameter can be varied between zero and unity. The desired delay is achievable by properly multiplying the outputs of $G_p(z)$ for $p = 0, 1, \ldots, P$ by $(1-2\mu)^l$ for $l = 1, 2, \ldots, L$, as shown in Fig. 1.

2.2. Filter Characteristics

For the given value of μ, the overall transfer function is expressible as

$$H(\Phi, z, \mu) = \sum_{n=0}^{2M-1} h(\Phi, n, \mu) z^{-n}, \tag{2a}$$

where

$$h(\Phi, n, \mu) = \sum_{l=0}^{L} \widehat{g}_l(\Phi, n)(1-2\mu)^l \quad \text{with} \quad \widehat{g}_l(\Phi, n) = \begin{cases} g_0(\Phi, n) & \text{for } l = 0 \\ g_1(\Phi, n) & \text{for } l = 1 \\ \quad \vdots & \\ g_{P-2}(\Phi, n) & \text{for } l = P-2 \\ \sum_{p=0}^{\lceil P/2-1 \rceil} \alpha_{2p+1}^{(l)} g_{2p+1}(n) & \text{for } l \text{ odd and } l \geq P \\ \sum_{p=0}^{\lfloor P/2 \rfloor} \alpha_{2p}^{(l)} g_{2p}(n) & \text{for } l \text{ even and } l \geq P \end{cases} \tag{2b}$$

and Φ is the parameter vector containing the adjustable parameters of the subfilters $G_p(z)$ for $p = 0, 1, \ldots, P-2$.

The magnitude and phase delay responses of the structure of Fig. 1 are given by

$$|H(\Phi, e^{j\omega}, \mu)| = \left| \sum_{n=0}^{2M-1} h(\Phi, n, \mu) e^{-j\omega n} \right| \quad \text{and} \quad \tau_p(\Phi, \omega, \mu) = -\arg H(\Phi, e^{j\omega}, \mu)/\omega, \tag{3}$$

respectively. Here, $\arg H(\Phi, e^{j\omega}, \mu)$ denotes the unwrapped phase response of the overall filter.

3. Statement of the Problem

In this section the overall design criteria are given. In addition, this section states the optimization problem under consideration.

3.1. Filter Specifications

The above structure does not enable one to keep the magnitude response, as given by the first subequation in Eq. (3), within the given limits $1 \pm \delta_a$ and the phase delay response, as given by the second subequation in Eq. (3), within the limits $M-1+\mu \pm \delta_p$ in the overall baseband $[0,\ \pi]$. Therefore, this contribution concentrates on achieving the desired performance on the frequency band given by $\Omega_p = [0,\ \omega_p]$ for $\omega_p < \pi$.

The goal is find the values for the adjustable parameters to meet the following criteria:

$$\Delta_p = \max_{0 \le \mu < 1} \Big[\max_{\omega \in \Omega_p} |\tau_p(\Phi, \omega, \mu) - (M-1+\mu)| \Big] \le \delta_p \quad \text{and} \quad \Delta_a = \max_{0 \le \mu < 1} \Big[\max_{\omega \in \Omega_p} \big| |H(\Phi, e^{j\omega}, \mu)| - 1 \big| \Big] \le \delta_a. \tag{4}$$

If the above criteria are met, then for each value of $\mu \in [0,\ 1]$, the magnitude and the phase delay responses stay for $\omega \in \Omega_p$ within the limits $1 \pm \delta_a$ and $M - 1 + \mu \pm \delta_p$, respectively.

3.2. Optimization Problem

The optimization problem under consideration is the following: Given $\Omega_p = [0,\ \omega_p]$, δ_a, and δ_p, find the adjustable parameters of the proposed structure to minimize

$$\epsilon = \max\{\Delta_p/\delta_p, \Delta_a/\delta_a\}. \tag{5}$$

Here, Δ_p and Δ_a are given by Eq. (4). Selecting the quantity to be minimized according to Eq. (5) is motivated by the fact that, in this case, the resulting relative error values, Δ_p/δ_p and Δ_a/δ_a, with respect to the given values become the same for both the phase delay and magnitude errors.

4. Filter Optimization

The solution to the stated optimization problem can be found in three steps. First, a straightforward design scheme is used for estimating the subfilter orders and other parameters. The second step involves finding the initial coefficient values for all the subfilters as well as for the additional coefficients. This initial filter is used as a start-up filter for further optimization being carried out by a constrained nonlinear optimization algorithm. In the third step, the following experimentally observed facts are utilized:

Observation 1. Some coefficients $g_p(n)$ have a negligible effect on the overall system performance and can be fixed to be zero-valued.

Observation 2. After fixing some coefficient values to be equal to zero according *Observation 1*, there exist some values of n for which the sums of the values of the all the existing $g_p(n)$'s in Fig. 1 for p even or p odd are very close to zero. Forcing these sums to be exactly equal to zero has again a very small effect on the overall system characteristics.

By properly exploiting *Observation 1* and *Observation 2* considerably reduces the implementation complexity and the parameters to be optimized.

Given $\Omega_p = [0, \omega_p]$, δ_a, and δ_p in the criteria stated by Eq. (4), the optimized filter is generated in the following three steps:

Step 1: Find the minimum value of M in such a way that the magnitude response of $G_0(z)$ of the resulting order $2M - 1$ approximates unity for $\omega \in \Omega_p$ such that the maximum deviation from unity is less than $\zeta\delta_a$. Here, $\zeta < 1$ is selected in a proper manner. For the reason to be described later, it has turned out that a good choice for ζ in most cases is within 0.3 and 0.7.

Step 2: Increase P, the number of filters, and L, the polynomial order, in the structure of Fig. 1 and design $G_p(z)$ for $p = 1, 2, \ldots, P-2$ as well as the additional coefficients $\alpha_p^{(l)}$ for $p = 0, 1, \ldots, P$ and for $l = P-1, P, \ldots, L$ until the criteria, as given by Eq. (4), are met by $\gamma\delta_a$ and $\gamma\delta_p$. It has turned out that a good selection for L is $L = P$ or $L = P+1$, whereas a good selection for γ is around 0.75.

Step 3: Exploit the above-mentioned *Observation 1* and *Observation 2* and re-optimize the remaining parameters to meet the given criteria, as given by Eq. (4).

In the above, the use of Step 1 is based on the fact that for the fractional delay μ equal to 0.5, the performance of the system of Fig. 1 is uniquely determined by $G_0(z)$. This is because for this value, $(1-2\mu)^l$ is zero except for $l = 0$. For this step, the minimum value of M can be directly determined by using the Remez multiple exchange algorithm by simply selecting one passband region equal to $\Omega_p = [0,\, \omega_p]$ [6].

The importance of Step 1 is the fact that it provides a simple way for determining the minimum odd order $2M-1$ for the subfilters in the overall system of Fig. 1. Because determining $G_0(z)$ at Step 1 corresponds to designing the overall system only for $\mu = 0.5$, the passband ripple of $G_0(z)$ should be smaller than δ_a that is included in the overall magnitude criterion, as given by the first subequation in Eq. (4). It has been experimentally observed that selecting the ripple of $G_0(z)$ to be within $0.3\delta_a$ and $0.7\delta_a$ enables one to generate the overall system in the above synthesis scheme.

Step 2 can be accomplished as follows: First, $G_p(z)$ for $p = 1, 2, \ldots, P-2$ of the same order $2M-1$ are designed using the Remez multiple exchange algorithm [11] in such a manner that the zero-phase frequency response of $G_p(z)$ approximates for $\omega \in \Omega_p$ the response of pth-order differentiator as given by

$$D_p(\omega) = \frac{(-1)^{\lfloor 3p/2 \rfloor}\omega^p}{p!2^p}. \tag{6}$$

This is due to the fact that for the adjustable fractional delay FIR filters based on the use of the Farrow structure the magnitude response of $G_p(z)$ approximates the magnitude response of pth-order differentiator, as given by Eq. (6) [10]. Second, the additional coefficients $\alpha_l^{(p)}$ are optimized in such a manner that maximum absolutely error between the realized and the desired frequency response, as given by

$$\max_{0 \le \mu < 1}\left[\max_{\omega \in \Omega_p}\left|H(\Phi, e^{j\omega}, \mu) - e^{-j\omega(M-1+\tau)}\right|\right] \le \delta, \tag{7}$$

is minimized. This problem can be solved using the real rotation theorem and linear programming [12]. Finally, the coefficients of the subfilters as well as the additional parameters are simultaneously optimized using a nonlinear optimization algorithm for minimizing ϵ, as given by Eq. (5).

When performing Step 3 the number of unknowns to be optimized is reduced in the manner to be described next. *Observation 1* can be utilized based on the following experimentally observed facts that will be considered in more detail in connections with the examples of Section 5. For $p \ge 1$, some of the $g_p(n)$'s for $n < M-1$ have a negligible effect on the overall system performance. These coefficients can be determined by gradually fixing the values of $g_p(n)$'s to be equal to zero and re-optimizing the rest of the coefficients until their effects on increasing both the worst-case magnitude and phase delay errors are still tolerable. *Observation 2* can utilized as follows. First, after exploiting *Observation 1*, it is checked whether for some values of n, the sums of the values of all the existing $g_p(n)$'s for p even or for p odd are close to zero. Second, these sums are gradually fixed to be exactly equal to zero and the rest of the coefficients are re-optimized until the resulting maximum magnitude and phase delay errors are still acceptable compared with the given error tolerances. It has turned out that over-designing the infinite-precision overall system at Step 3 with tolerances being 75 percent of the given ones provides enough margins for both exploiting *Observation 1* and *Observation 2*.

5. Examples

This section illustrates, by means of examples taken from the literature, the flexibility and the efficiency of the proposed optimization scheme. In addition, the performance and the complexity of the proposed filters are compared with other FIR adjustable fractional delay filters.

5.1. Example 1

It is required that $\Omega_p = [0,\, 0.9\pi]$, $\delta_a = 0.01$, and $\delta_p = 0.001$ [4, 6, 10]. When performing Step 1 in the proposed synthesis scheme, the minimum odd order for which the magnitude response of $G_0(z)$ stays within $1 \pm \zeta\delta_a$ on Ω_p becomes 27 ($M = 14$).

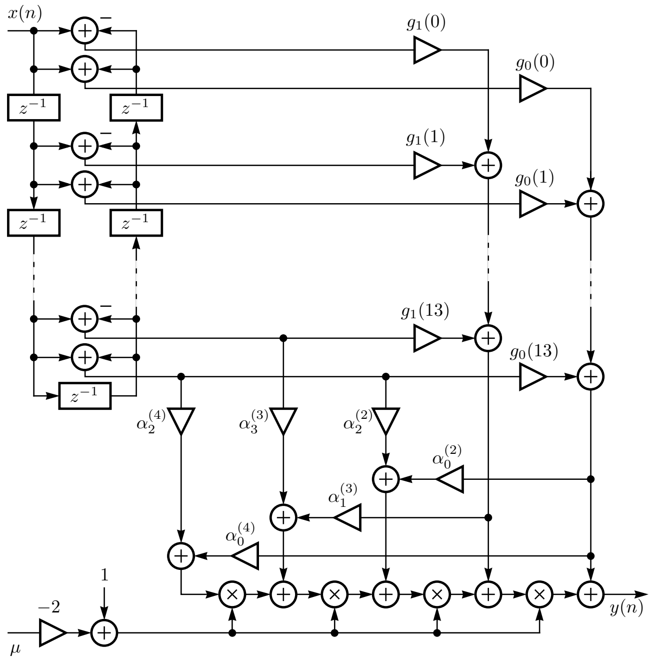

Fig. 2 Efficient implementation of the adjustable fractional delay filter in Example 1 for the $M = 14$, $P = 3$, and $L = 4$ case.

Table 1. Optimized coefficient values in Example 1 for the $M = 14$, $P = 4$, and $L = 4$ case.

$G_0(z)$		$G_1(z)$		$G_2(z)$	
$g_0(0) = -0.002\,63$	$g_0(7) = 0.029\,78$	$g_1(0) = 0$	$g_1(7) = 0$	$g_2(0) = 0$	$g_2(7) = -g_0(7)$
$g_0(1) = 0.003\,08$	$g_0(8) = -0.043\,38$	$g_1(1) = 0$	$g_1(8) = 0$	$g_2(1) = 0$	$g_2(8) = 0.052\,52$
$g_0(2) = -0.006\,07$	$g_0(9) = 0.059\,98$	$g_1(2) = 0$	$g_1(9) = 0.003\,94$	$g_2(2) = -g_0(2)$	$g_2(9) = -0.067\,03$
$g_0(3) = 0.008\,90$	$g_0(10) = -0.079\,67$	$g_1(3) = 0$	$g_1(10) = -0.010\,21$	$g_2(3) = -g_0(3)$	$g_2(10) = 0.093\,52$
$g_0(4) = -0.012\,36$	$g_0(11) = 0.120\,69$	$g_1(4) = 0$	$g_1(11) = 0.021\,80$	$g_2(4) = -g_0(4)$	$g_2(11) = -0.140\,00$
$g_0(5) = 0.016\,67$	$g_0(12) = -0.207\,42$	$g_1(5) = 0$	$g_1(12) = -0.066\,72$	$g_2(5) = -g_0(5)$	$g_2(12) = 0.229\,32$
$g_0(6) = -0.022\,23$	$g_0(13) = 0.635\,75$	$g_1(6) = 0$	$g_1(13) = 0.632\,53$	$g_2(6) = -g_0(6)$	$g_2(13) = -0.148\,59$
$\alpha_1^{(3)} = -1$	$\alpha_3^{(3)} = 0.497\,19$	$\alpha_0^{(4)} = -1$	$\alpha_2^{(4)} = -1$	$\alpha_4^{(4)} = 0.496\,83$	

For this design, the maximum deviation from unity is $0.005\,020$. At Step 2, for the $P = L = 3$ design, Δ_a, the worst-case magnitude error, and Δ_p, the worst-case phase delay error, are $\Delta_a = 10\Delta_p = 0.030\,351$.

The minimum values of P and L required to meet the given overall criteria become three and four, respectively, that is, the overall structure of Fig. 1 consists four parallel filters of order $2M - 1 = 27$. For the optimized overall design, $\Delta_a = 10\Delta_p = 0.009\,182$. Even though the margins are small compared to the given criteria, *Observation 1* can be exploited by fixing $g_1(0)$, $g_1(1)$ and $g_1(8)$ to be zero-valued resulting in $\Delta_a = 10\Delta_p = 0.009\,969$. In this case, *Observation 2* can not be utilized due to the reason that the overall filter has only four parallel filters. An efficient implementation for the optimized adjustable fractional delay filter sharing the delay elements between $G_p(z)$ for $p = 0, 1, 2, 3$ is depicted in Fig. 2.

The minimum implementation complexity is obtained by selecting $P = L = 4$. For the optimized overall design, $\Delta_a = 10\Delta_p = 0.006\,687$. When exploiting *Observation 1*, it is noticed that $g_1(n)$ for $n = 0, 1, \ldots, 8$ and $g_2(n)$ for $n = 0, 1$ have a negligible effect on the overall filter performance. If these coefficients are fixed to be zero-valued, then re-optimizing the overall system gives $\Delta_a = 10\Delta_p = 0.008\,664$. In addition, the values of $g_2(n)$ and $-g_0(n)$ for $n = 2, 3, \ldots, 7$ are practically the same. Forcing these values to be equal, that is, exploiting *Observation 2*, results in $\Delta_a = 10\Delta_p = 0.009\,742$. The optimized coefficient values are shown in Table 1.

This example has also been considered in [4, 6, 10]. For the optimized system in [4] meeting the same criteria the number of subfilters is five $(L = 4)$ and the subfilter order is 25 $(M = 13)$. The overall number of multipliers is 69 when including the general multipliers required to implement the multiplications of the outputs of $G_l(z)$ by $(1 - 2\mu)^l$ for $l = 0, 1, \ldots, 4$ whereas

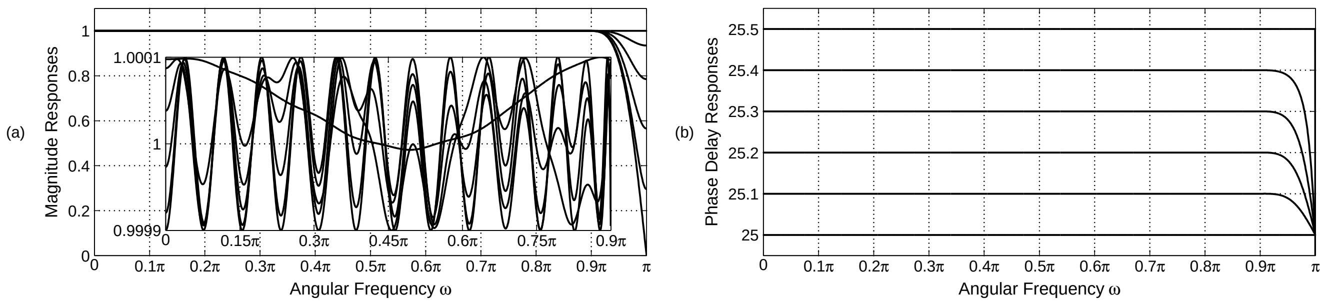

Fig. 3 Frequency responses for the optimized overall filter for some values of μ in Example 2. (a) Magnitude response as well as the passband details. (b) Phase delay responses.

Table 2. Summary of filter designs in Examples 1 and 2.

	M	L	P	$\Delta_a = 10\Delta_p$	N_A	N_M
Modified Farrow [4]	13	4	–	0.006 571	91	69
Modified Farrow [10]	14	5	–	0.005 608	80	57
Modified Farrow [6]	14	4	–	0.009 069	56	32
Proposed	14	4	3	0.009 969	56	35
Proposed	14	4	4	0.009 742	50	31
	M	L	P	$\Delta_a = \Delta_p$	N_A	N_M
Modified Farrow [6]	26	6	–	$9.927 \cdot 10^{-5}$	136	73
Proposed	26	6	5	$9.986 \cdot 10^{-5}$	107	74

the overall number of adders and subtracters is 91. In [10], six subfilters ($L = 5$) of order 27 ($M = 14$) is used to meet the specifications. In addition, $g_1(n) = 0$ for $n = 0, 1, \ldots, 7$, $g_3(n) = 0$ for $n = 0, 1, \ldots, 6$, $g_4(n) = 0$ for $n = 0, 1, 2$, and $g_5(n) = 0$ for $n = 0, 1, \ldots, 9$. The overall number of multipliers is 57 as reported in [10]. For this design, the number of adders and subtracters required for the implementation is 80 For the filter optimized in [6], the overall number of multipliers and the number of adders and subtracters are 32 and 56, respectively. In this case, five subfilters of order 25 are used to satisfy the specifications, whereas $g_1(n) = g_3(n) = 0$ for $n = 0, 1, \ldots, 8$, $g_4(n) = 0$ for $n = 0, 1 \ldots, 6$, $g_1(n) + g_3(n) = 0$ for $n = 9, 10, 11, 12$, and $g_0(n) + g_2(n) + g_4(n) = 0$ for $n = 7, 8, \ldots, 12$. For the best proposed design, the corresponding figures are 31 and 50.

5.2. Example 2

It is required that $\Omega_p = [0, 0.9\pi]$ and $\delta_a = \delta_p = 10^{-4}$. In this case, the specifications are met by $M = 26$, $P = 5$, and $L = 6$. The worst-case magnitude error and the worst-case phase error for the optimized design are $\Delta_p = \Delta_a = 8.282 \cdot 10^{-5}$. The coefficients $g_1(n)$ and $g_3(n)$ for $n = 0, 1, \ldots, 14$ have a negligible effect on the overall system performance. In addition, when fixing $g_2(n) = -g_0(n)$ for $n = 0, 1, \ldots, 9$ and $g_3(n) = -g_1(n)$ for $n = 14, 15, \ldots, 19$, $\Delta_p = \Delta_a = 9.986 \cdot 10^{-5}$ is achievable. The number of multipliers for the proposed design is 74, whereas the number of adders and subtracters is 107. For the filter optimized utilizing the technique proposed in [6], the corresponding figures are 73 and 136. The magnitude and phase delay responses for the optimized design are shown for some values of μ in Figs. 2(a) and 2(b), respectively. Since for μ and $1 - \mu$ the magnitude and phase delay distortions are the same, only the values of μ in the range $[0, 0.5]$ have to be considered.

5.3. Summary

The summary of the filter designs in Examples 1 and 2 are shown in Table 2. In this table, Δ_a and Δ_p denote the worst-case deviation of the magnitude response from the unity and the worst-case deviation of the phase delay response from $M - 1 + \mu$ on Ω_p, respectively, whereas N_A and N_M denote, respectively, the number of adders and subtracters and multipliers required for the overall implementation.

Acknowledgment

This work was supported by the Academy of Finland, project No. 44876 (Finnish Centre of Excellence Program (2000–2005)). Juha Yli-Kaakinen was also financed by a postdoctoral research grant from the Academy of Finland, project No. 105823.

References

[1] C. W. Farrow, "A continuously variable digital delay element," in *Proc. IEEE Int. Symp. Circuits Syst.*, Espoo, Finland, June 7–9 1988, pp. 2641–2645.

[2] T. I. Laakso, V. Välimäki, M. Karjalainen, and U. K. Laine, "Splitting the unit delay," *IEEE Signal Process. Mag.*, vol. 13, no. 1, pp. 30–60, Jan. 1996.

[3] J. Vesma and T. Saramäki, "Interpolation filters with arbitrary frequency response for all-digital receivers," in *Proc. IEEE Int. Symp. Circuits Syst.*, Atlanta, GA, May 12–15 1996, pp. 568–571.

[4] J. Vesma and T. Saramäki, "Optimization and efficient implementation of FIR filters with adjustable fractional delay," in *Proc. IEEE Int. Symp. Circuits Syst.*, vol. 1, Hong Kong, June 9–12 1997, pp. 2256–2259.

[5] J. Yli-Kaakinen and T. Saramäki, "Multiplier-free polynomial-based FIR filters with an adjustable fractional delay," in *Proc. IEEE Int. Conf. Electr. Circuits Syst. (ICECS 2002)*, vol. III, Dubrovnik, Croatia, Sept. 15–18 2002, pp. 1167–1170. [Online]. Available: http://alpha.cc.tut.fi/˜ylikaaki/ICECS02.pdf

[6] J. Yli-Kaakinen and T. Saramäki, "Multiplier-free polynomial-based FIR filters with an adjustable fractional delay," to appear in *Circuits, Systems, and Signal Processing*, 2006.

[7] M. Makundi, T. I. Laakso, and V. Välimaki, "Efficient tunable IIR and allpass filter structures," *Electron. Lett.*, vol. 37, no. 6, pp. 344–345, Mar. 2001.

[8] C.-C. Tseng, "Design of 1-D and 2-D variable fractional delay allpass filters using weighted least-squares method," *IEEE Trans. Circuits Syst. I*, vol. 49, no. 10, pp. 1413 –1422, Oct. 2002.

[9] J. Yli-Kaakinen and T. Saramäki, "An algorithm for the optimization of adjustable fractional-delay all-pass filters," in *Proc. IEEE Int. Symp. Circuits Syst.*, vol. III, Vancouver, Canada, May 23–26 2004, pp. 153–156. [Online]. Available: http://www.cs.tut.fi/˜ylikaaki

[10] H. Johansson and P. Löwenborg, "On the design of adjustable fractional delay FIR filters," *IEEE Trans. Circuits Syst. II*, vol. 50, pp. 164–169, Apr. 2003.

[11] C. Rahenkamp and B. V. Kumar, "Modifications to the McClellan, Parks, and Rabiner computer program for designing higher order differentiating FIR filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 34, no. 6, pp. 1671–1674, Dec. 1986.

[12] R. L. Streit and A. H. Nuttall, "A general Chebyshev complex function approximation procedure and an application to beamforming," *J. Acoust. Soc. Amer.*, vol. 72, no. 1, pp. 181–190, July 1982.