A Simplified Structure for FIR Filters with an Adjustable Fractional Delay

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***Abstract*— This paper introduces an efficient filter structure for implementing finite-impulse response (FIR) filters with an adjustable fractional delay. In this structure the first two subfilters are the same as in the modified Farrow structure, whereas the remaining ones are generated by properly combining these two subfilters with some additional very short filters, pure delay terms, adders, and multipliers. For significantly reducing the number of multipliers, the three-step synthesis scheme proposed by Yli-Kaakinen and Saramäki in the case of the modified Farrow structure is followed. First, the number of subfilters and their orders are determined such that the given criteria are sufficiently exceeded. Second, an initial filter is determined using a simple design scheme. This filter serves as a start-up solution for further optimization being performed using a constrained nonlinear optimization algorithm. Third, those coefficient values of the subfilters having a negligible effect on the overall system performance are fixed to be zero-valued. Both the performance and complexity of the proposed adjustable digital filters are compared with those of some existing adjustable FIR filters proposed in the literature. This comparison shows that, in the case of stringent amplitude and phase delay specifications, the number of multipliers for the proposed filters is less than 80 percent when compared with the corresponding optimized modified Farrow structure.**

I. Introduction

IN VARIOUS digital signal processing applications, there is a need for a delay that is a fraction of the sampling interval. Furthermore, it is often desired that the delay value is adjustable or variable during the computation. These applications include, among others, sampling rate conversion, echo cancellation, phased array antenna systems, time delay estimation, timing adjustment in all-digital receivers, modeling of musical instruments, and speech coding and synthesis [1], [2].

Adjustable fractional-delay filters can be designed either using finite-impulse response (FIR) or infinite-impulse response (IIR) filters. One computationally efficient technique, belonging to the former filter class, is to use the Farrow structure [1] consisting of several parallel fixed FIR filters. Recently, the modified Farrow structure has been introduced by Vesma and Saramäki in [3], [4] by properly modifying the original structure. The modified structure consists of a given number of fixed linear-phase FIR filters of the same odd order and the impulse-response coefficients alternatively possess an even and odd symmetry such that the first filter has a symmetrical impulse response. The desired fractional delay of the value μ is achieved by multiplying the outputs of these filters with quantities directly depending on this value of μ [1], [2], [4], [5]. Another attractive class of adjustable fractional delay filters, belonging to the second class, are adjustable fractional delay all-pass filters based on the use of the so-called gathering structure [2], [6]–[8] proposed by Makundi, Laakso, and Välimäki. In this structure, the filter coefficients are polynomials of the fractional delay parameter.

The purpose of this paper is twofold. First, an efficient structure is proposed for implementing FIR filters with an adjustable fractional delay. Second, the synthesis scheme proposed by Yli-Kaakinen and Saramäki [5] for designing filters for the modified Farrow structure is applied to the proposed structure. In the proposed structure, the two first branch filters are the same as in the modified Farrow structure,

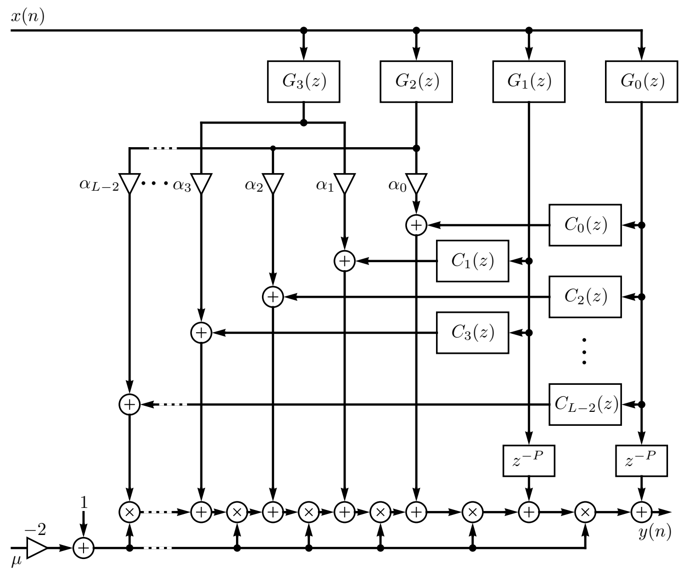

Fig. 1. Proposed structure with an adjustable fractional delay μ. The impulse-response coefficients of the $G_l(z)$'s alternatively possess an even and odd symmetry such that the first filter has a symmetrical impulse response.

whereas the rest of them are generated with aid of these branch filters and two simple filters by adding their outputs cascaded by some very short filters. As in the case of the modified Farrow structure, the computational complexity of the proposed structure can be significantly reduced by using the synthesis scheme proposed in [5].

II. Proposed Structure with an Adjustable Fractional Delay

This section introduces the proposed structure for implementing FIR filters with an adjustable fractional delay. In addition, the magnitude and phase delay responses of this structure are given for the later use.

A. Overall Filter Structure

The proposed structure consisting of four parallel FIR filters for generating an adjustable fractional delay μ is depicted in Fig. 1. The transfer functions of the two first FIR filters are of the form

$$G_{0,1}(z) = \sum_{n=0}^{M-1} g_0(n)z^{-n} \pm \sum_{n=M}^{2M-1} g_0(2M-1-n)z^{-n}, \quad \text{(1a)}$$

respectively. The transfer function with the plus sign and with the minus sign are the transfer functions of $G_0(z)$ and $G_1(z)$, respectively, whereas the transfer functions of the remaining two FIR filters are given in a same manner by

$$G_{2,3}(z) = z^{-(M+P-1)} \pm z^{-(M+P)}, \quad \text{(1b)}$$

respectively. Here, M and P are integers and thus the orders of all these filters are odd. In addition, the proposed structure contains $L-1$ additional short subfilters of order $2P$, as given by

$$C_l(z) = \sum_{n=0}^{P} c_l(n) z^{-n} + \sum_{n=P+1}^{2P} c_l(2P-n) z^{-n} \tag{1c}$$

for $l = 0, 1, \ldots, L-2$.

After optimizing the impulse-response coefficients of the above subfilters as well as the additional coefficients α_l for $l = 0, 1, \ldots, L-2$, as shown in Fig. 1, in the manner to be described later on, the role of the adjustable parameter μ in Fig. 1 is to generate the delay approximating $M + P - 1 + \mu$ in the given passband region. This parameter can be varied between zero and unity. The desired delay is achievable by properly multiplying the outputs of the branch filters, as given by (2c), by $(1-2\mu)^l$ for $l = 1, 2, \ldots, L$, as shown in Fig. 1.

B. Filter Characteristics

For the given value of μ, the overall transfer function is expressible as

$$H(\Phi, z, \mu) = \sum_{n=0}^{2M-1} h(\Phi, n, \mu) z^{-n}, \tag{2a}$$

where

$$h(\Phi, n, \mu) = \sum_{l=0}^{L} \widehat{g}_l(\Phi, n)(1-2\mu)^l \tag{2b}$$

with

$$\widehat{g}_l(\Phi, n) = \begin{cases} z^{-P} g_0(n), & \text{for } l = 0 \\ z^{-P} g_1(n), & \text{for } l = 1 \\ c_{l-2}(n) * g_0(n) + \alpha_{l-2} g_2(n), & \text{for } l \text{ even and } l \neq 0 \\ c_{l-2}(n) * g_1(n) + \alpha_{l-2} g_3(n), & \text{for } l \text{ odd and } l \neq 1 \end{cases} \tag{2c}$$

and Φ is the parameter vector containing the adjustable parameters of the subfilters $G_l(z)$ for $l = 0, 1, 2, 3$ and $C_l(z)$ for $l = 0, 1, \ldots, L-2$ as well as additional coefficients α_l for $l = 0, 1, \ldots, L-2$.

The magnitude and phase delay responses of the structure of Fig. 1 are given by

$$|H(\Phi, e^{j\omega}, \mu)| = \left| \sum_{n=0}^{2M-1} h(\Phi, n, \mu) e^{-j\omega n} \right| \tag{3a}$$

and

$$\tau_p(\Phi, \omega, \mu) = -\arg H(\Phi, e^{j\omega}, \mu)/\omega, \tag{3b}$$

respectively. Here, $\arg H(\Phi, e^{j\omega}, \mu)$ denotes the unwrapped phase response of the overall filter.

III. Statement of the Problem

In this section the overall design criteria are given. In addition, this section states the optimization problem under consideration.

A. Filter Specifications

The above structure does not enable one to keep the magnitude response, as given by (3a), within the given limits $1 \pm \delta_a$ and the phase delay response, as given by (3b), within the limits $M+P-1+\mu\pm\delta_p$ in the overall baseband $[0, \pi]$. Therefore, this contribution concentrates on achieving the desired performance on the frequency band given by $\Omega_p = [0, \omega_p]$ with $\omega_p < \pi$.

The goal is find the values for the adjustable parameters to meet the following criteria:

$$\Delta_p = \max_{0 \le \mu < 1} \left[\max_{\omega \in \Omega_p} |\tau_p(\Phi, \omega, \mu) - (M + P - 1 + \mu)| \right] \le \delta_p \tag{4a}$$

and

$$\Delta_a = \max_{0 \le \mu < 1} \left[\max_{\omega \in \Omega_p} ||H(\Phi, e^{j\omega}, \mu)| - 1| \right] \le \delta_a. \tag{4b}$$

If the above criteria are met, then for each value of $\mu \in [0, 1]$, the magnitude and the phase delay responses stay for $\omega \in \Omega_p$ within the limits $1 \pm \delta_a$ and $M + P - 1 + \mu \pm \delta_p$, respectively.

B. Optimization Problem

The optimization problem under consideration is the following: Given $\Omega_p = [0, \omega_p]$, δ_a, and δ_p, find the adjustable parameters of the proposed structure to minimize

$$\epsilon = \max\{\Delta_p/\delta_p, \Delta_a/\delta_a\}. \tag{5}$$

Here, Δ_p and Δ_a are given by (4a) and (4b). Selecting the quantity to be minimized according to (5) is motivated by the fact that, in this case, the resulting relative error values, Δ_p/δ_p and Δ_a/δ_a, with respect to the given values become the same for both the phase delay and magnitude errors.

IV. Filter Optimization

The solution to the stated optimization problem can be found in three steps. First, a straightforward design scheme is used for estimating the subfilter orders and other parameters. The second step involves finding the initial coefficient values for all the subfilters as well as for the additional coefficients. This initial filter is used as a start-up filter for further optimization being carried out by a constrained nonlinear optimization algorithm. In the third step, the following experimentally observed fact is utilized:

Observation: Some coefficients $g_1(n)$ and the $c_l(n)$'s have a negligible effect on the overall system performance and can be fixed to be zero-valued.

By properly exploiting the above mentioned *Observation* considerably reduces the implementation complexity and the parameters to be optimized.

Given $\Omega_p = [0, \omega_p]$, δ_a, and δ_p in the criteria stated by (4a) and (4b), the optimized filter is generated in the following three steps:

Step 1: Find the minimum value of M in such a way that the magnitude response of $G_0(z)$ of the resulting order $2M-1$ approximates unity for $\omega \in \Omega_p$ such that the maximum deviation from unity is less than $\zeta\delta_a$. Here, $\zeta < 1$ is selected in a proper manner. For the reason to be described later, it has turned out that a good choice for ζ in most cases is within 0.5 and 0.7.

Step 2: Design $G_1(z)$ and increase $2P$, the order of the $C_l(z)$'s, and $L+1$, the overall number of branch filters, in the structure of Fig. 1 and design $C_l(z)$ for $l = 0, 1, \ldots, L-2$ as well as the additional coefficients α_l for $l = 0, 1, \ldots, L-2$ until the criteria, as given by (4a) and (4b), are met by $\gamma\delta_a$ and $\gamma\delta_p$. It has turned out that a good selection for γ is around 0.75.

Step 3: Exploit the above-mentioned *Observation* and re-optimize the remaining parameters to meet the given criteria, as given by (4a) and (4b).

In the above, the use of Step 1 is based on the fact that for the fractional delay μ equal to 0.5, the performance of the system of

Fig. 1 is uniquely determined by $G_0(z)$. This is because for this value, $(1-2\mu)^l$ is zero except for $l = 0$. For this step, the minimum value of M can be directly determined by using the Remez multiple exchange algorithm by simply selecting one passband region equal to $\Omega_p = [0,\ \omega_p]$ [5].

The importance of Step 1 lies in the fact that it provides a simple way for determining the minimum odd order $2M-1$ for the subfilters in the overall system of Fig. 1. Because determining $G_0(z)$ at Step 1 corresponds to designing the overall system only for $\mu = 0.5$, the passband ripple of $G_0(z)$ should be smaller than δ_a that is included in the overall magnitude criterion, as given by (4a). It has been experimentally observed that selecting the ripple of $G_0(z)$ to be within $0.5\delta_a$ and $0.7\delta_a$ enables one to generate the overall system in the above synthesis scheme.

Step 2 can be accomplished as follows: First, $G_1(z)$ of the same order $2M-1$ is designed using the Remez multiple exchange algorithm [9] in such a manner that the frequency response of $G_1(z)$ approximates for $\omega \in \Omega_p$ the response of first-order differentiator. This is due to the fact that for the adjustable fractional delay FIR filters based on the use of the Farrow structure the frequency response of $G_l(z)$ approximates the frequency response of lth-order differentiator [10]. Second, $C_l(z)$ for $l = 0, 1, \ldots, L-2$ and the additional coefficients α_l for $l = 0, 1, \ldots, L-2$ are optimized in such a manner that maximum absolutely error between the resulting and the desired frequency responses, as given by

$$\xi_l = \max_{\omega \in \Omega_p} \left| G_n(\Phi, e^{j\omega}) C_l(\Phi, e^{j\omega}) + \alpha_l G_m(\Phi, e^{j\omega}) - D_{l+2}(e^{j\omega}) \right| \tag{6a}$$

with

$$D_l(e^{j\omega}) = \frac{(-1)^{\lfloor 3l/2 \rfloor}\omega^l}{l!\,2^l}(-j)^{l^2} e^{-j(M+P-0.5)\omega} \tag{6b}$$

is minimized. Here $G_n(\Phi, e^{j\omega})$ and $G_m(\Phi, e^{j\omega})$ are $G_0(\Phi, e^{j\omega})$ and $G_2(\Phi, e^{j\omega})$ for l even, respectively, and $G_1(\Phi, e^{j\omega})$ and $G_3(\Phi, e^{j\omega})$ for l odd, respectively. This problem can be solved using semidefinite programming. The minimum order for the $C_l(z)$'s can be determined by gradually increasing the common order $2P$ until the maximum absolutely error between the resulting and the desired frequency responses, as given by (6a) and (6b), is of the same order of magnitude as $\gamma\delta_a$ at Step 2 of the algorithm of Section IV. Finally, the coefficients of the subfilters as well as the additional parameters are simultaneously optimized using a nonlinear optimization algorithm for minimizing ϵ, as given by (5).

When performing Step 3 the number of unknowns to be optimized is reduced in the manner to be described next. *Observation* can be utilized based on the following experimentally observed fact that will be considered in more detail in connections with the examples of Section V. Some of the $g_1(n)$'s for $n < M-1$ and $c_l(n)$'s for $n < P$ have a negligible effect on the overall system performance. These coefficients can be determined by gradually fixing the values of these $g_1(n)$'s and $c_l(n)$'s to be equal to zero and re-optimizing the rest of the coefficients until their effects on increasing both the worst-case magnitude and phase delay errors are still tolerable. It has turned out that over-designing the infinite-precision overall system at Step 3 with tolerances being 75 percent of the given ones provides enough margins for exploiting *Observation*.

V. Examples

This section illustrates, by means of examples taken from the literature, the flexibility and the efficiency of the proposed optimization scheme. In addition, the performance and the complexity of the proposed filters are compared with other FIR adjustable fractional delay filters.

TABLE I
Optimized Coefficient Values in Example 1

$G_0(z)$		$G_1(z)$	
$g_0(0) = g_0(27) =$	$-0.003\,1608$	$g_1(0) = -g_1(27) =$	0
$g_0(1) = g_0(26) =$	$0.004\,1784$	$g_1(1) = -g_1(26) =$	0
$g_0(2) = g_0(25) =$	$-0.005\,5910$	$g_1(2) = -g_1(25) =$	0
$g_0(3) = g_0(24) =$	$0.009\,1121$	$g_1(3) = -g_1(24) =$	0
$g_0(4) = g_0(23) =$	$-0.012\,8665$	$g_1(4) = -g_1(23) =$	0
$g_0(5) = g_0(22) =$	$0.017\,0865$	$g_1(5) = -g_1(22) =$	0
$g_0(6) = g_0(21) =$	$-0.023\,2003$	$g_1(6) = -g_1(21) =$	0
$g_0(7) = g_0(20) =$	$0.031\,1193$	$g_1(7) = -g_1(20) =$	0
$g_0(8) = g_0(19) =$	$-0.041\,6802$	$g_1(8) = -g_1(19) =$	0
$g_0(9) = g_0(18) =$	$0.060\,1425$	$g_1(9) = -g_1(18) =$	$0.003\,5235$
$g_0(10) = g_0(17) =$	$-0.080\,4449$	$g_1(10) = -g_1(17) =$	$-0.010\,2502$
$g_0(11) = g_0(16) =$	$0.120\,1248$	$g_1(11) = -g_1(16) =$	$0.022\,9843$
$g_0(12) = g_0(15) =$	$-0.207\,7476$	$g_1(12) = -g_1(15) =$	$-0.066\,2902$
$g_0(13) = g_0(14) =$	$0.636\,5536$	$g_1(13) = -g_1(14) =$	$0.632\,0982$
$c_0(0) = c_0(2) =$	$-0.006\,2320$	$c_0(1) =$	$-1.167\,7135$
$c_1(0) = c_1(2) =$	0	$c_1(1) =$	$-0.997\,1582$
$c_2(0) = c_2(2) =$	0	$c_2(1) =$	$0.148\,5866$
$\alpha_0 = 0.597\,8638$	$\alpha_1 = 0.494\,7460$	$\alpha_2 = -0.086\,6984$	

A. Example 1

It is required that $\Omega_p = [0,\ 0.9\pi]$, $\delta_a = 0.01$, and $\delta_p = 0.001$ [4], [5], [10]. When performing Step 1 in the proposed synthesis scheme of Section IV, the minimum odd order for which the magnitude response of $G_0(z)$ stays within $1 \pm \zeta\delta_a$ on Ω_p becomes 27 ($M = 14$). For this design, the maximum deviation from unity is $0.005\,020$. At Step 2, for the $P = 0$ and $P = 1$ designs $\xi_0 = 0.028\,043$ and $\xi_0 = 0.008\,148$ [cf. (6a) and (6b)], respectively, that is, the minimum order for the $C_l(z)$'s becomes two. For the $L = 3$ design, Δ_a, the worst-case magnitude error, and Δ_p, the worst-case phase delay error, become $\Delta_a = 10\Delta_p = 0.030\,443$. This indicates that L should be increased.

The minimum value of L required to meet the given overall criteria become four, that is, the overall structure of Fig. 1 consists three filters, $C_l(z)$ for $l = 0, 1, 2$ of order $2P = 2$. For the optimized overall design, $\Delta_a = 10\Delta_p = 0.006\,825$. When exploiting *Observation*, it is noticed that $g_1(n)$ for $n = 0, 1, \ldots, 8$ and $c_l(0)$ for $l = 1, 2$ have a negligible effect on the overall filter performance. If these coefficients are fixed to be zero-valued, then re-optimizing the overall system gives $\Delta_a = 10\Delta_p = 0.009\,594$. Thus the number of multipliers for the overall filter required to meet the given specifications becomes 30. The optimized coefficient values are shown in Table I.

This example has also been considered in [4], [5], [10]. For the optimized conventional Farrow structure in [4] meeting the same criteria, the number of subfilters is five ($L = 4$) and the subfilter order is 25 ($M = 13$). The overall number of multipliers is 69 when including the general multipliers required to implement the multiplications of the outputs of $G_l(z)$ by $(1-2\mu)^l$ for $l = 0, 1, \ldots, 4$. In [10], six subfilters ($L = 5$) of order 27 ($M = 14$) is used to meet the specifications. In addition, $g_1(n) = 0$ for $n = 0, 1, \ldots, 7$, $g_3(n) = 0$ for $n = 0, 1, \ldots, 6$, $g_4(n) = 0$ for $n = 0, 1, 2$, and $g_5(n) = 0$ for $n = 0, 1, \ldots, 9$. The overall number of multipliers is 57 as reported in [10]. For the filter optimized in [5], the overall number of multipliers is 32.

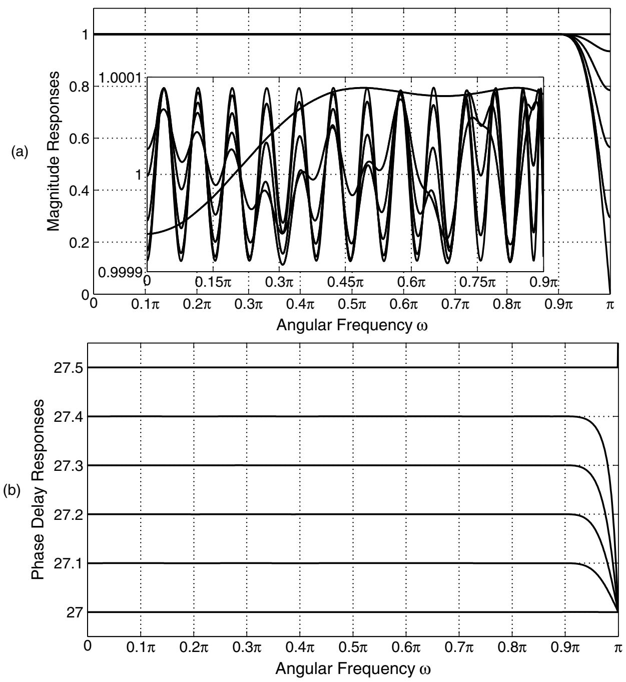

Fig. 2. Frequency responses for the optimized overall filter for some values of μ in Example 2. (a) Magnitude response as well as the passband details. (b) Phase delay responses.

TABLE II
SUMMARY OF THE FILTER DESIGNS IN EXAMPLES 1, 2, AND 3

	M	L	P	$\Delta_a = 10\Delta_p$	N_{M}
Modified Farrow [4]	13	4	–	0.006 571	69
Modified Farrow [10]	14	5	–	0.005 608	57
Modified Farrow [5]	14	4	–	0.009 069	32
Proposed	14	4	1	0.009 501	30
	M	L	P	$\Delta_a = \Delta_p$	N_{M}
Modified Farrow [5]	26	6	–	$9.927 \cdot 10^{-5}$	73
Proposed	26	6	2	$8.901 \cdot 10^{-5}$	60
Modified Farrow [10]	38	10	–	$\approx 10^{-5}$	250
Modified Farrow [11]	25	13	–	$\approx 10^{-5}$	227
Proposed	34	9	3	$7.816 \cdot 10^{-6}$	100

B. Example 2

It is required that $\Omega_p = [0, 0.9\pi]$ and $\delta_a = \delta_p = 10^{-4}$. In this case, the specifications are met by $M = 26$, $P = 2$, and $L = 6$. The worst-case magnitude error and the worst-case phase error for the optimized design are $\Delta_p = \Delta_a = 8.282 \cdot 10^{-5}$. The coefficients $g_1(n)$ for $n = 0, 1, \ldots, 14$ have a negligible effect on the overall system performance. In addition, when fixing $c_1(0) = 0$, $c_3(0) = 0$, and $c_4(0) = 0$, $\Delta_p = \Delta_a = 9.986 \cdot 10^{-5}$ is achievable. The number of multipliers for the proposed design is 60, whereas the number of adders and subtracters is 107. For the filter optimized utilizing the technique proposed in [5], the corresponding figures are 73 and 136. The magnitude and phase delay responses for the optimized proposed design are shown for some values of μ in Figs. 2(a) and 2(b), respectively. Since for μ and $1 - \mu$ the magnitude and phase delay distortions are the same, only the values of μ in the range $[0, 0.5]$ have to be considered.

C. Example 3

The specifications are the same as in Example 1 except that both the maximum allowable magnitude error and the maximum allowable phase delay are now $\delta_a = \delta_p = 10^{-5}$ [10], [11]. In this case, the specifications are met by $M = 34$, $P = 3$, and $L = 9$. The worst-case magnitude error and the worst-case phase error for the optimized design are $\Delta_p = \Delta_a = 7.816 \cdot 10^{-6}$. The number of multipliers for the proposed design is 100. For the filters optimized in [10] and [11], the corresponding figures are 250 and 227, respectively. It should be noted that the objective function for the filters optimized in [10], [11] and for the proposed ones are different. In [10], [11], the objective is to minimize the maximum absolute value of the complex error between the desired and the resulting responses.

D. Summary

The summary of the filter designs in Examples 1, 2, and 3 are shown in Table II. In this table, Δ_a and Δ_p denote the worst-case deviation of the magnitude response from the unity and the worst-case deviation of the phase delay response from $M + P - 1 + \mu$ on Ω_p, respectively, whereas N_{M} denotes the number of multipliers required for the overall implementation.

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