Multiplier-Free Polynomial-Based FIR Filters with an Adjustable Fractional Delay

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ABSTRACT

An efficient coefficient quantization scheme is described for minimizing the cost for implementing the fixed parallel linear-phase finite-impulse response (FIR) filters in the modified Farrow structure introduced by Vesma and Saramäki for generating FIR filters with an adjustable fractional delay. The implementation costs under consideration are the minimum number of adders and/or subtracters in two cases required in the overall implementation to meet the given overall criteria. In the first case, the coefficients are implemented independently of each others, whereas in the second case, the common subexpressions within the coefficients are shared in order to reduce the implementation cost even further. The optimum finite-precision solution is found in four steps. First, the number of filters and their lengths are determined such that the given criteria are sufficiently exceeded in order to allow some coefficient quantization errors. Second, those impulse-response values of the subfilters having a negligible effect on the overall system performance are fixed to be zero-valued. Third, constrained nonlinear optimization is applied to determining for the remaining infinite-precision coefficients a parameter space including the feasible space where the given criteria are met. The fourth step involves finding in this space the finite-precision coefficient values for minimizing the given implementation cost. Examples are included illustrating the efficiency of the proposed synthesis scheme.

1. INTRODUCTION

RECENTLY, the modified Farrow structure shown in Fig. 1 has been introduced by Vesma and Saramäki in [1]–[3] for generating finite-impulse response (FIR) filters with an adjustable fractional delay. This structure has been developed by properly modifying the original structure proposed by Farrow in [4]. The modified structure contains a given number of fixed linear-phase FIR filters of the same even length and the impulse-response coefficients alternatively possess an even and odd symmetry such that the first filter has a symmetrical impulse response. Another attractive feature of this structure is that the (even) lengths of the fixed FIR filters as well as the number of filters can be arbitrarily selected. Furthermore, the desired fractional delay can be achieved by properly multiplying the outputs of these filters with quantities depending directly on the value of the fractional delay. Originally, the modified Farrow structure for generating FIR filters with an adjustable fractional delay has been developed by using a model where the discrete input sequence is first converted into its analog equivalent with the aid of the ideal digital-to-analog converter and a polynomial-based reconstruction filter. Then, the desired delay is generated by delaying the output of this analog filter before the analog-to-digital conversion [2]. This is the reason for the title of this paper.

The purpose of this contribution is to describe an efficient scheme for quantizing the coefficients of the fixed filter sections in such a way that the cost is minimized in two cases for implementing the fixed filter sections subject to the condition that the given criteria are still met. In the first case, all the filter coefficients are separately quantized to be a few powers of two and the implementation cost is the number of adders (or subtracters) required to implement all the fixed filters. In the second case, the common subexpressions within the coefficients are shared in order to reduce the implementation cost even further [5]. The main difference compared to the implementation of a single linear-phase FIR filter is that for the modified Farrow structure there are several parallel FIR filters and the common subexpressions are shared with all these filters in the case where the fixed filters can be implemented using the transposed direct-form structure (exploiting the coefficient symmetry), as proposed in [6].

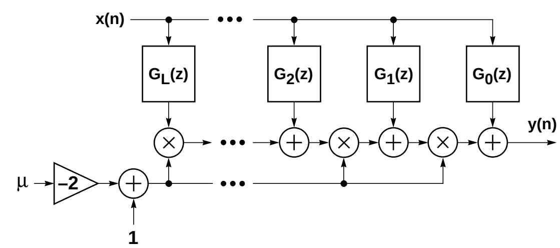

Fig. 1. The modified Farrow structure with an adjustable fractional delay μ.

The above-mentioned goals are achieved in four steps. In the first step, the required (even) filter lengths and the number of branch filters are determined in a predetermined manner in such a way that the infinite-precision solution meets the given criteria for all values of the fractional delay between zero and unity sufficiently well in order to allow the desired coefficient quantization errors. In the second step, those impulse-response coefficients having a negligible effect on the overall system performance are set to be zero-valued. Simultaneously, this reduces the implementation cost and the number of coefficients to be quantized. The third step is based on the following observation made by the authors of this paper in [7] when quantizing the coefficients of a conventional linear-phase FIR filter. In order to find the best finite-precision solution, it has turned out to be very beneficial to first find the smallest and largest values for all the filter coefficients in such a way that the given criteria are still met by re-optimizing the remaining coefficient values. This enables one to find a parameter space which includes the feasible space where the specifications are satisfied. After finding this larger space, all what is needed is to check whether in this space there exist the desired discrete values for the coefficient representations. In performing the third and fourth steps, several observations made on the performance of the fixed filters in the modified Farrow structure are utilized to simplify and to make the overall coefficient quantization scheme faster.

2. MODIFIED FARROW STRUCTURE WITH AN ADJUSTABLE FRACTIONAL DELAY

This section briefly reviews the modified Farrow structure proposed by Vesma and Saramäki in [1]–[3] for generating FIR filters with an adjustable fractional delay. In addition, the amplitude and phase delay responses as well as the overall design criteria are given for the later use.

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2.1. Overall Filter Structure

The proposed structure with an adjustable fractional delay μ is depicted in Fig. 1 [3]. It consists of $L+1$ parallel FIR filters with transfer functions of the form

$$G_l(z) = \sum_{n=0}^{N-1} g_l(n) z^{-n} \quad \text{for } l = 0, 1, \ldots, L, \tag{1}$$

where N is an even integer. The impulse-response coefficients $g_l(n)$ for $n = 0, 1, \ldots, N/2-1$ fulfill the following symmetry conditions:

$$g_l(n) = \begin{cases} g_l(N-1-n) & \text{for } l \text{ even} \\ -g_l(N-1-n) & \text{for } l \text{ odd.} \end{cases} \tag{2}$$

After optimizing the above impulse-response coefficients in the manner to be described later on, the role of the adjustable parameter μ in Fig. 1 is to generate the delay equal to $N/2-1+\mu$ in the given passband region. This parameter can be varied between zero and unity. The desired delay is achievable by multiplying the output of $G_l(z)$ by $(1-2\mu)^l$ for $l = 0, 1, \ldots, L$, as shown in Fig. 1.

2.2. Filter Characteristics

For the given value μ, the overall transfer function is expressible as

$$H(\Phi, z, \mu) = \sum_{n=0}^{N-1} h(\Phi, n, \mu) z^{-n}, \tag{3a}$$

where

$$h(\Phi, n, \mu) = \sum_{l=0}^{L} g_l(n)(1-2\mu)^l \tag{3b}$$

and Φ is the adjustable parameter vector given by

$$\begin{aligned} \Phi = \big[& g_0(0), \ldots, g_0(N/2-1), g_1(0), \ldots, \\ & g_1(N/2-1), \ldots, g_L(0), \ldots, g_L(N/2-1) \big]. \end{aligned} \tag{3c}$$

The frequency, amplitude, and phase delay responses of the structure of Fig. 1 are given by

$$H(\Phi, e^{j\omega}, \mu) = \sum_{n=0}^{N-1} h(\Phi, n, \mu) e^{-j\omega n}, \tag{4a}$$

$$|H(\Phi, e^{j\omega}, \mu)| = \Big| \sum_{n=0}^{N-1} h(\Phi, n, \mu) e^{-j\omega n} \Big|, \tag{4b}$$

and

$$\tau_p(\Phi, \omega, \mu) = -\arg H(\Phi, e^{j\omega}, \mu)/\omega, \tag{4c}$$

respectively.

2.3. Filter Specifications

The above structure does not enable one to keep the amplitude response, as given by Eq. (4b), within the given limits $1 \pm \delta_a$ and the phase delay response, as given by Eq. (4c), within the limits $N/2-1+\mu \pm \delta_p$ in the overall baseband $[0,\ \pi]$. Therefore, this contribution concentrates on achieving the desired performance on the frequency band given by

$$\Omega_p = [0,\ \omega_p], \quad \omega_p < \pi. \tag{5}$$

The goal is to meet with infinite-precision or finite-precision coefficient values the following criteria:

$$\Delta_p = \max_{0 \le \mu < 1} \Big[\max_{\omega \in \Omega_p} |\tau_p(\Phi, \omega, \mu) - (N/2-1+\mu)| \Big] \le \delta_p \tag{6a}$$

and

$$\Delta_a = \max_{0 \le \mu < 1} \Big[\max_{\omega \in \Omega_p} \big| |H(\Phi, e^{j\omega}, \mu)| - 1 \big| \Big] \le \delta_a. \tag{6b}$$

When the above criteria are met, then for each value of μ, the amplitude and the phase delay responses stay within the limits $1 \pm \delta_a$ and $N/2-1+\mu \pm \delta_p$ on Ω_p.

3. STATEMENT OF THE PROBLEM

This section states the problem for meeting the criteria of Eq. (6) with the minimized implementation cost of the subfilter transfer functions $G_l(z)$ for $l = 0, 1, \ldots, L$.

3.1. Transfer Function under Consideration

In the sequel, it is assumed that each adjustable filter coefficient $g_l(n)$ for $l = 0, 1, \ldots, L$ and $n = 0, 1, \ldots, N/2-1$ in the structure of Fig. 1 is expressed as a sums of signed-powers-of-two (SPT) terms as follows:

$$g_l(n) = \sum_{k=1}^{W_n^{(l)}+1} a_{k,n}^{(l)} 2^{-P_{k,n}^{(l)}}, \tag{7}$$

where $a_{k,n}^{(l)} \in \{-1, 1\}$ and $P_{k,n}^{(l)} \in \{1, 2, \ldots, M\}$ for $k = 1, 2, \ldots, W_n^{(l)}+1$. In this representation form, each coefficient $g_l(n)$ has $W_n^{(l)}$ adders (or subtracters) and the maximum allowable wordlength is M bits.

When finding the optimized simple discrete-value representation forms for the impulse-response coefficients of FIR filters, it is a common practice to accomplish the optimization in such a manner that the scaled response meets the given amplitude criteria [8], [9]. In this case, the criterion, as given by Eq. (6b), is replaced by

$$\Delta_a = \max_{0 \le \mu < 1} \Big[\max_{\omega \in \Omega_p} \big| |H(\Phi, e^{j\omega}, \mu)/\beta| - 1 \big| \Big] \le \delta_a, \tag{8}$$

where β is an additional adjustable parameter. These criteria are preferred to be used when the filter coefficients are desired to be quantized on a highly nonuniform discrete grid as in the case of the power-of-two coefficients. In this case, is it beneficial to take the passband gain of the filter as an optimization variable together with the filter coefficients. Obviously, this is not a problem since, in many applications, the major concern is the relative stopband attenuation (i.e., relative to passband attenuation) and the desired level for the passband amplitude response is achieved, if desired, by using the scaling constant equal to $1/\beta$.

3.2. Implementation Cost

This contribution concentrates on designing multiplierless transfer functions $G_l(z)$ for $l = 0, 1, \ldots, L$ in two cases. In the first case, the coefficients are implemented independently of each others, that is, the redundancy within the coefficients is not utilized. The goal is to find all the filter coefficient values in such a manner that the filter meets the given specifications and the implementation cost for the filter is minimized. In the second case, the common subexpressions within the coefficients are shared in order to reduce the implementation cost even further [5].

To estimate the overall implementation cost, we consider the number of adders and subtracters required to implement all the coefficients as well as the number of structural adders, that is, all the adders at the filter output and those inside the delay line for the direct-form and transposed direct-form implementations, respectively. This is because the shifts can be implemented almost for free using hard-wired shifters and the number of adders (or subtracters) required to implement all the coefficients depends on the filter implementation, e.g., whether the coefficient symmetry is exploited and whether the redundancy within the coefficients is utilized. If the coefficient symmetry is exploited, then a reasonable estimate for the cost for implementing the fixed filters in Fig. 1 can be expressed as

$$(L+1)(N-2) - 2Q + \sum_{l=0}^{L} \Bigg[\sum_{n=0}^{N/2-1} W_n^{(l)} \Bigg], \tag{9}$$

where $W_n^{(l)}$ is the number of adders required to implement $g_l(n)$ [see Eq. (7)] and Q is the number of zero-valued coefficients requiring no implementation ($W_n^{(l)} = 0$). For the case where the subexpression elimination is exploited, there exists no simple formula for the implementation cost.

3.3. Optimization Problem

The optimization problem under consideration is the following:

Optimization Problem: Given Ω_p, δ_a, δ_p, and M, the number of fractional bits, find the adjustable parameter vector, as given by Eq. (3c), and β to minimize the given implementation cost in such a manner that, first, the criteria given by Eqs. (6a) and (8) are met and, then,

$$\max\{\Delta_p/\delta_p, \Delta_a/\delta_a\}, \tag{10}$$

is minimized. Here, Δ_p and Δ_a are given by Eqs. (6a) and (8), respectively.

4. FILTER OPTIMIZATION

The solution to the stated optimization problem can be found in three steps. In the first step, the system of Fig. 1 with infinite-precision coefficients is determined in such a way that it exceeds the given criteria to provide some tolerance for the coefficient quantization. In the second step, a parameter space is generated including the feasible space where the infinite-precision system meets the given criteria. The third step involves finding in this space the desired finite-precision coefficients.

4.1. Generating the Start-Up Filter

Given Ω_p, δ_a, and δ_p, the start-up filter can be generated in the following three steps:

Step 1: Find the minimum even order N such that the optimized amplitude response of $G_0(z)$ in the structure of Fig. 1 approximates unity on Ω_p such that the maximum deviation from unity is less than $\alpha\delta_a$. Here, $\alpha < 1$ is selected in a proper manner. It has turned out that a good choice for α in many cases is within 0.5 and 0.75.

Step 2: Increase $L + 1$, the number of filters in the structure of Fig. 1 until the criteria given by Eq. (6) are met by $\gamma\delta_a$ and $\gamma\delta_p$. A good selection for γ is around 0.75.

Step 3: Set those coefficients of the filters in the structure of Fig. 1 equal to zero that have a negligible effect on the overall system performance.

In the above, Step 1 is based on the fact that for the fractional delay μ equal to half, the performance of the system of Fig. 1 is uniquely determined by $G_0(z)$. This is because for this value, $(1-2\mu)^l$ is zero except for $l = 0$. For this step, the minimum value of N can be determined directly by using the Remez multiple exchange algorithm by simply selecting one passband region equal to Ω_p.[1]

Step 2 can be accomplished in a manner described in [3]. When performing Step 3 the only exception is that some of the unknowns are disregarded. For later use, the remaining adjustable parameter vector with the disregarded unknowns is denoted by $\hat{\Phi}$.

4.2. Feasible Parameter Space for the Infinite-Precision Filter Structure

When generating a parameter space for infinite-precision coefficients including the feasible space where the infinite-precision filter meets the given criteria it is beneficial to mimic the scheme discovered by the authors of this paper in [7] for quantizing the coefficients of a single linear-phase FIR filter. Based on this discovery, a very straightforward quantization scheme for the coefficients is obtained as follows. For each remaining filter coefficient $g_l(n)$ included in $\hat{\Phi}$, the smallest and largest values of the coefficient are determined in such a manner that the given amplitude criteria are met subject to $g_0(N/2-1) = 1$.[2] This restriction, simplifying the overall procedure, can be stated without loss of generality since the scaling constant β included in Eq. (8) can be used for achieving the desired passband amplitude level. If J is the number of remaining coefficients in the parameter vector $\hat{\Phi}$, the goal is achieved by solving $2(J-1)$ ($g_0(N/2-1) \equiv 1$) problems of the following form. For each remaining coefficient $g_l(n)$, find the minimum and maximum value by optimizing the parameter vector $\hat{\Phi}$ and β such the criteria of Eqs. (6a) and (8) are met.

In order to solve the above problems, we discretize the passband region into the frequency points $\omega_r \in \Omega_p = [0, \omega_p]$, $r = 1, 2, \ldots, R$ and the range $0 \leq \mu \leq 1/2$ into the points $\mu_s \in [0, 1/2]$, $s = 1, 2, \ldots, S$.[3] In many cases, $R = 20N$ and $S = 100$ are good selections to arrive at a very accurate solution. The desired discretized problems are of the following form: Find $\hat{\Phi}$ and β to minimize ψ subject to conditions

$$g_0(N/2-1) = 1 \tag{11a}$$

as well as

$$|\tau_p(\hat{\Phi}, \omega_r, \mu_s) - (N/2 - 1 + \mu_s)| - \delta_p \leq 0 \tag{11b}$$

and

$$||H(\hat{\Phi}, e^{j\omega_r}, \mu_s)/\beta| - 1| - \delta_a \leq 0 \tag{11c}$$

for $r = 1, 2, \ldots, R$ and $s = 1, 2, \ldots, S$.

For each remaining coefficient $g_l(n)$, the minimum and maximum achievable value is obtained by solving the above optimization problem for both $\psi = g_l(n)$ and $\psi = -g_l(n)$. Various techniques for solving the above problem can be found in [10].

4.3. Optimization of Finite-Precision Coefficients

It has been experimentally observed that the parameter space defined above includes the feasible space where the filter specifications are satisfied. After finding this larger space, all what is needed is to check whether in this space there exists a combination of the discrete coefficient values with which the overall criteria are met. This can performed in a manner similar to that used in [7] for quantizing the coefficients of a single linear-phase FIR filter. The main differences are that now there are several parallel linear-phase filters, instead of a single filter, and $g_0(N/2-1)$ plays the same role as $h(M)$ in [7].

5. EXAMPLES

This section illustrates, by means of examples, the flexibility and effectiveness of the proposed optimization scheme.

5.1. Example 1

It is required that $\Omega_p = [0,\ 0.75\pi]$, $\delta_a = 0.01$, and $\delta_p = 0.01$. The minimum even order for which the amplitude response of $G_0(z)$ stays within $1 \pm \delta_p$ on Ω_p is 12. For this design the maximum deviation from unity is 0.00391. The next step is to determine the minimum L in such a way that the given overall criteria are met. The resulting value is three so that overall structure of Fig. 1 consists four parallel filters of length 12. If Δ_a, the worst-case amplitude error, and Δ_p, the worst-case phase delay error, are fixed to be the same, then for the optimized system $\Delta_a = \Delta_p = 0.0051$. The coefficients $g_1(n)$ and $g_3(n)$ for $n = 0, 1, 2, 3$ have a negligible effect on the overall system performance. If these coefficients are fixed to be zero-valued, then the filter optimization gives $\Delta_a = \Delta_p = 0.0055$. Furthermore, when fixing $g_2(n) = -g_0(n)$ for $n = 0, 1, \ldots, 4$ and $g_3(4) = -g_1(4)$ $\Delta_a = \Delta_p = 0.0069$ is achievable. The optimized coefficients for this design are shown in Table 1. Table 2, in turn, gives the optimized finite-precision coefficients in the case where the subexpression elimination between the filter coefficients is performed. In order to utilize this, all the parallel filters should be implemented using the transposed direct-form structure (exploiting the coefficient symmetry). The overall number of adders and subtracters in implementing all the coefficients is six. When including the structural adders this figure increases by 28. The amplitude and phase delay responses for the optimized finite-precision filter are shown for some values of μ in Figs. 2

[1] Since $N-1$, the filter order, is odd the overall filter has a fixed zero at $z = -1$. Furthermore, for this filter the delay is identically equal to $N/2 - 1 + 1/2$.

[2] This coefficient value is always present after disregarding those coefficients having a negligible effect on the overall system performance.

[3] Since for μ and $1-\mu$ the amplitude and phase delay distortions are the same, only the values of μ in the range $[0,\ 1/2]$ have to be considered.

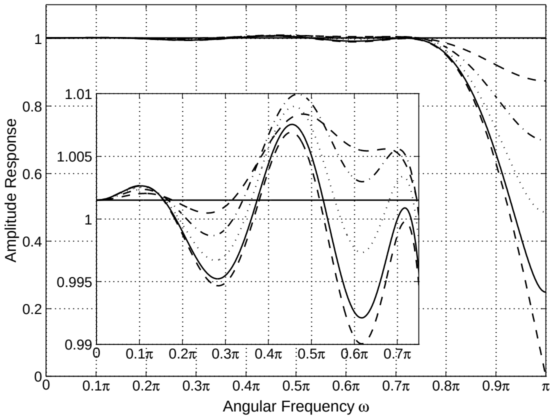

Fig. 2. Amplitude responses for the optimized finite-precision filter of Example 1 ($N = 12$ and $L = 3$) as well as the passband details for $\mu = 0,\ 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5$. The dashed line with the value of zero at $\omega = \pi$, solid line with a lower value at $\omega = \pi$, dotted line, dot-dashed line, dashed line with a higher value at $\omega = \pi$, and solid line with a higher value at $\omega = \pi$ give the responses for $\mu = 0.5$, for $\mu = 0.4$, for $\mu = 0.3$, for $\mu = 0.2$, for $\mu = 0.1$, and for $\mu = 0$, respectively.

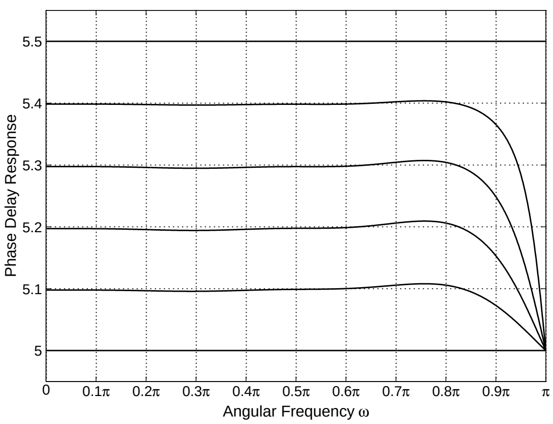

Fig. 3. Phase delay responses for the optimized finite-precision filter of Example 1 ($N = 12$ and $L = 3$) for $\mu = 0,\ 0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5$.

and 3. Since for μ and $1 - \mu$ the amplitude and phase delay distortions are the same, only the values of μ in the range $[0,\ 1/2]$ have to be considered.

5.2. Example 2

It is required that $\Omega_p = [0,\ 0.75\pi]$, $\delta_a = 0.025$, and $\delta_p = 0.005$. Table 3 shows the optimized finite-precision coefficients in the case where the subexpression elimination between the filter coefficients is performed. The overall number of adders and subtracters in implementing all the coefficients is five. For the filter optimized in [6] for meeting approximately the same criteria the corresponding figure is 18. Furthermore, for this and the proposed design the number structural adders are 42 and 24, respectively.

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Table 1. Optimized infinite-precision coefficient values for the filter in Example 1 ($N = 12$ and $L = 3$)

$g_1(n) = g_3(n) = 0$ for $n = 0, 1, 2, 3$	
$g_0(0) = -0.0096115$	$g_2(0) = -g_0(0)$
$g_0(1) = \ \ 0.0213330$	$g_2(1) = -g_0(1)$
$g_0(2) = -0.0453523$	$g_2(2) = -g_0(2)$
$g_0(3) = \ \ 0.0896136$	$g_2(3) = -g_0(3)$
$g_0(4) = -0.1873974$	$g_2(4) = -g_0(4)$
$g_0(5) = \ \ 0.6279599$	$g_2(5) = -0.1314137$
$g_1(4) = -0.0405611$	$g_3(4) = -g_1(4)$
$g_1(5) = \ \ 0.6004100$	$g_3(5) = -0.0977848$

Table 2. Optimized finite-precision coefficient values for the filter in Example 1 ($N = 12$ and $L = 3$)

$g_1(n) = g_3(n) = 0$ for $n = 0, 1, 2, 3$	
$g_a = -2^{-2} + 2^{-4}$	$g_b = 2^{-1} + 2^{-3}$
$g_0(0) = -2^{-7}$	$g_2(0) = -g_0(0)$
$g_0(1) = -2^{-3} \cdot g_a$	$g_2(1) = -g_0(1)$
$g_0(2) = \ \ 2^{-2} \cdot g_a$	$g_2(2) = -g_0(2)$
$g_0(3) = -2^{-1} \cdot g_a$	$g_2(3) = -g_0(3)$
$g_0(4) = \ \ g_a - 2^{-7}$	$g_2(4) = -g_0(4)$
$g_0(5) = \ \ g_b + 2^{-5}$	$g_2(5) = -2^{-3} - 2^{-7}$
$g_1(4) = -2^{-5}$	$g_3(4) = -g_1(4)$
$g_1(5) = \ \ g_b$	$g_3(5) = \ \ 2^{-1} \cdot g_a - 2^{-7}$

Table 3. Optimized finite-precision coefficient values for the filter in Example 2 ($N = 10$ and $L = 3$)

$g_1(n) = g_3(n) = 0$ for $n = 0, 1, 2$	
$g_a = -2^{-3} - 2^{-5}$	
$g_0(0) = \ \ 2^{-6}$	$g_2(0) = -g_0(0)$
$g_0(1) = -2^{-5}$	$g_2(1) = -g_0(1)$
$g_0(2) = \ \ 2^{-4} + 2^{-7}$	$g_2(2) = -g_0(2)$
$g_0(3) = \ \ g_a$	$g_2(3) = -g_0(3)$
$g_0(4) = \ \ 2^{-1} + 2^{-5}$	$g_2(4) = -2^{-3} + 2^{-5}$
$g_1(3) = -2^{-5}$	$g_3(3) = -g_1(3)$
$g_1(4) = \ \ 2^{-1} + 2^{-6}$	$g_3(4) = \ \ 2^{-1} \cdot g_a$

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