Frequency-Response Masking Based FIR Filter Design with Power-of-Two Coefficients and Optimum PWR

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ABSTRACT

This paper presents a new method for designing sharp linear phase FIR filters with power-of-two coefficients. The method is based on a frequency-response masking technique. In this method, the power-of-two coefficients and continuous scaling parameters of the subfilters are taken to be decision variables, and peak weighted ripple(PWR) is taken to be the design objective. The resulting nonlinear mixed integer optimization problem for each subfilter is first reduced to an equivalent discrete optimization problem whose search region is then cropped for efficiency of computation, similar to the approach in [8], although different subregions are used here. The effectiveness of the method is demonstrated through a low pass linear phase sharp FIR digital filter example.

1. INTRODUCTION

It is demonstrated in [4, 5, 6] that the frequency-response masking(FRM) technique is an efficient method for reducing the complexity of a sharp FIR filter. The basic structure of a filter synthesized using the frequency-response masking technique is shown in Figure 1. If $F(z)$ is the z-transform transfer function of the system, then, from Figure 1,

$$F(z) = F_a(z^M) \cdot F_{Ma}(z) + F_c(z^M) \cdot F_{Mc}(z).$$

Here, $F_a(z^M)$ is obtained by replacing each delay element of a prototype filter, $F_a(z)$, by M delay elements. $F_c(z^M)$ is the complement of $F_a(z^M)$, and $F_{Ma}(z)$, $F_{Mc}(z)$ are the masking filters of the system. The frequency response magnitudes of $F_a(z^M)$, $F_c(z^M)$, $F_{Ma}(z)$ and $F_{Mc}(z)$ are shown in Figure 2. If $F_a(z^M) \cdot F_{Ma}(z)$ and $F_c(z^M) \cdot F_{Mc}(z)$ have the same phase delay, the resulting frequency response magnitude of $F(z)$ is shown in Figure 2 (c). The transition width of $|F(e^{jw})|$ is narrower than that of $|F_a(e^{jw})|$ by a factor of M.

The coefficients of each subfilter are assumed to lie in a power-of-two space. The search region for these coefficients is reduced to a combination of several subregions. The search for the optimal coefficients of each subfilter takes place over each subregion. These searches can be carried out independently, thus allowing the use of parallel processing.

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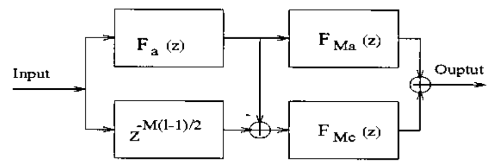

Fig. 1. The structure for a filter synthesized using the frequency response masking technique

2. THE PROBLEM

The general expression of a FIR filter coefficient, $h(n)$, as a sum of signed power-of-two (SPT) terms is given by

$$h(n) = \sum_{i=1}^{p_n} s_{k,n} 2^{g_{k,n}},$$

where $s_{k,n} \in \{-1, 1\}$ and $g_{k,n} \in \{0, 1, \cdots, b-1\}$ for $k = 1, 2, \cdots, p_n$. That is, each coefficient, $h(n)$, has p_n SPT terms and b-bit coefficient wordlength. The total number of SPT terms of a filter is constrained by

$$\sum_{i=1,}^{l} p_i \leq N. \quad (1)$$

Here, l is the number of the coefficients of the filter. l and the total number of SPT terms, N, may vary from subfilter to subfilter.

Given an impulse response up-sampling ratio, M, the passband edge, w_p, and stopband edge, w_s, of a desired filter, the band edges for the subfilters of the system, F_a, F_{Ma} and F_{Mc}, can be derived and the optimal lengths of the filters can be estimated (for details, see [4, 5]). Therefore, the general problem of designing a FIR filter using the frequency-response masking technique may be formulated as: given an

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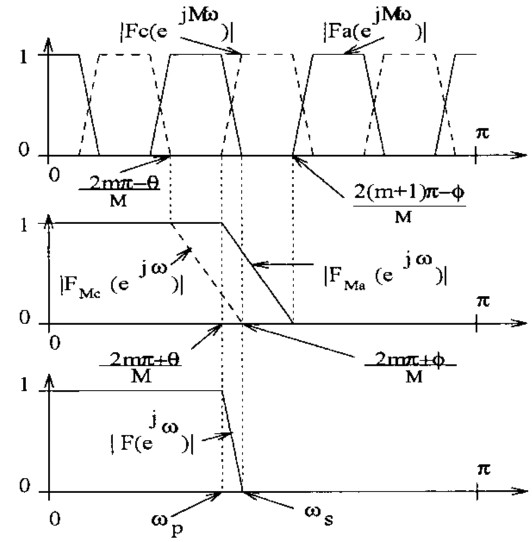


Fig. 2. Frequency responses of $F_a(z^M)$, $F_{Ma}(z)$, $F_{Mc}(z)$ and $F(z)$. Here, θ and ϕ are passband edge and stopband edge of $F_a(z)$, respectively.

impulse response up-sampling ratio, M, and the band edges and filter lengths of the subfilters of the system, find coefficients, $\boldsymbol{h}_a$, $\boldsymbol{h}_{Ma}$, $\boldsymbol{h}_{Mc}$ of the filters F_a, F_{Ma}, F_{Mc}, respectively, to minimize the peak weighted ripple (PWR) of the system, given by

$$e(\boldsymbol{h}_a, \boldsymbol{h}_{Ma}, \boldsymbol{h}_{Mc}) = \max_{f\in\{\mathcal{P}\cup\mathcal{S}\}} W(w)|D(w) - F(w)|.$$

Here, $\mathcal{P}$ and $\mathcal{S}$ are the passband and stopband of the system, respectively. $W(w)$ and $D(w)$ are frequency weighting function and desired objective of the system, respectively. In this paper, $\boldsymbol{h}_a$, $\boldsymbol{h}_{Ma}$ and $\boldsymbol{h}_{Mc}$ are constrained to power-of-two space, and, together with scaling factors λ_a, λ_{Ma}, λ_{Mc} of the subfilters, F_a, F_{Ma} and F_{Mc}, are to be considered as decision variables.

3. SOLUTION METHOD

Let $G_a(w)$ and $\delta_a(w)$ be the desired value and deviation, respectively, of $F_a(e^{jMw})$. Further, let $G_{Ma}(w)$, $G_{Mc}(w)$, and $\delta_{Ma}(w)$, $\delta_{Mc}(w)$ be the desired values and deviations of $F_{Ma}(e^{jw})$ and $F_{Mc}(e^{jw})$, respectively. If $G(w)$ and $\delta(w)$ are, respectively, the desired value and deviation of $F(e^{jw})$, we have

$$\begin{aligned}G(w)+\delta(w)=&\{G_{Ma}(w)+\delta_{Ma}(w)\}\{G_a(w)+\delta_a(w)\}\\&+\{G_{Mc}(w)+\delta_{Mc}(w)\}\{G_c(w)+\delta_c(w)\}.\end{aligned}$$

It can be shown that where $G_{Ma}(w)$ and $G_{Mc}(w)$ are both equal to zero or one, $\delta(w)$ is determined primarily by $\delta_{Ma}(w)$ or $\delta_{Mc}(w)$ depending on whether $G_a(w)$ is zero or one. The effect of $\delta_a(w)$ is of secondary importance. Hence, F_{Ma} and F_{Mc} are filters with don't care bands within their passbands and stopbands. In these don't care bands, the relatively large deviation, δ_{Ma} (or δ_{Mc}), will not effect the PWR of the system.

It can also be shown that it is possible to design F_a so that δ_a partially compensates for $\delta_{Ma}(w)$ and $\delta_{Mc}(w)$ for those frequencies near the transition band of $F(w)$. Hence, F_{Ma} and F_{Mc} should be designed first. Then, F_a can be designed to compensate for $\delta_{Ma}(w)$ and $\delta_{Mc}(w)$ (for details, see. [4]).

3.1. Design of F_{Ma} and F_{Mc}

The problem for designing F_{Ma} (or F_{Mc}) can be formulated as: find $\boldsymbol{h}$ and λ, such that

$$e(\boldsymbol{h},\lambda) = \max_{w\in\{\mathcal{P}\cup\mathcal{S}\}} W(w)W_{Ma}(w)|D_{Ma}(w) - \lambda F_{Ma}(w)| \quad (2)$$

is minimized. Here, $\mathcal{P}$ and $\mathcal{S}$ are passband and stopband of the filter, F_{Ma}, respectively, $W(w)$ and $W_{Ma}(w)$ are the frequency weighting function for the design requirement and the don't care band of the filter, F_{Ma}, respectively, and $D_{Ma}(w)$ is the target response of the masking filter F_{Ma}. Let this optimization problem be referred to as Problem (P).

We now decompose Problem (P) into two levels as follows:
Problem (P_1): Given $\boldsymbol{h}$,

$$\min_{\lambda}\{e(\boldsymbol{h},\lambda)\}.$$

Let the solution obtained be denoted by $\lambda^*(\boldsymbol{h})$. Define

$$\bar{e}(\boldsymbol{h}) = e(\boldsymbol{h},\lambda^*(\boldsymbol{h})). \quad (3)$$

Let

$$D_{Ma}(w) = \begin{cases}1, & w\in\mathcal{P},\\ 0, & w\in\mathcal{S},\end{cases} \quad W(w) = \begin{cases}1, & w\in\mathcal{P},\\ \gamma, & w\in\mathcal{S}.\end{cases} \quad (4)$$

With respect to the solution, $\lambda^*(\boldsymbol{h})$, of Problem (P_1), we have the following theorem:

Theorem 3.1 *Let the functions $D_{Ma}(w)$, $W(w)$ of (2) be defined by (4), let $W_{Ma}(w) = 1$ and let γ be a constant. Then, for a given $\boldsymbol{h}$ which yields $\min_{w\in\mathcal{P}} F(w) > 0$, the Problem $(P_1(\boldsymbol{h}))$ has a positive solution $\lambda^*(\boldsymbol{h}) = \min\{\lambda_1, \lambda_2\}$, where $\lambda_1 = \frac{1}{\min_{w\in\mathcal{P}} F(w)+\gamma\max_{w\in\mathcal{S}}|F(w)|}$ and $\lambda_2 = \frac{2}{\min_{w\in\mathcal{P}} F(w)+\max_{w\in\mathcal{P}} F(w)}$.*

Proof: See [2].

Let Problem (P_2) be defined by

$$\min_{\boldsymbol{h}}\{\bar{e}(\boldsymbol{h})\}.$$

Problem (P_2) is a discrete optimization problem, which can be solved using a suitable global optimization approach such as simulated annealing, as discussed below.

We now propose to solve Problem (P) in two steps:

- Step 1. Find a reduced search region $\mathcal{A} = \mathcal{A}_1 \cup \cdots \cup \mathcal{A}_{m_1}$ for the discrete coefficients $\boldsymbol{h} = \{h(n), n = 1, \ldots l\}$. Here, m_1 is a small number (for example, 3).
- Step 2. Solve the Problem (P_2) over each of the reduced subregions $\mathcal{A}_i$, $i = 1, \cdots, m_1$, and then find the global minimum in the whole search region, $\mathcal{A}$.

3.1.1. Finding the Reduced Search Region $\mathcal{A}$

Obviously, when we apply a optimization method to Problem (P_2), the speed of convergence depends on the size of the feasible region. A smaller region will result in faster convergence. Thus, we propose the following scheme to reduce the search region:

1. Calculate the infinite precision coefficients, $\boldsymbol{h}_0$, of the masking filter, F_{Ma}, by using appropriate optimization methods such as linear Programming [7], and calculate the quantization scaling factor $q_0 = \max_{n \in \{1, \cdots, l\}} \left\{ \frac{|h_0(n)|}{2^b} \right\}$.
2. For each $q \in \mathcal{D} = \{q \in [q_0, 2q_0] : q = q_0 + n\Delta,\ n = 0, 1, \ldots, 2^{b+1},\ \Delta = \frac{q_0}{2^{b+1}}\}$, using the method proposed in [3], find a set of quantized coefficients, $\boldsymbol{h}_q = \{h_q(1), \cdots, h_q(l)\}$, satisfying the constraint (1), using the following steps:
   - Step 1: Let $h_q(i) = 0, i = 1, \cdots, l$, $\boldsymbol{E} = \boldsymbol{h}_0/q$, and set the iteration counter m=0.
   - Step 2: If $\|\boldsymbol{h}_q\|_\infty \leq 2^{-1}$, then stop. Otherwise find i such that $|h_q(i)| = \|\boldsymbol{h}_q\|_\infty$, and then find $s \in \{-1, 1\}$ and $j \in \{0, 1, \cdots, b-1\}$ such that $|E(i) - s2^j| = \min_{s,j} |E(i) - s2^j|$.
   - Step 3: $h_q(i) = h_q(i) + s2^j$, and $E(i) = E(i) - s2^j$.
   - Step 4: m=m+1. If m = N, as defined in (1), stop, otherwise go to Step2.
3. Using the algorithm obtained from Theorem 3.1 (with the modification that we replace the passband and stopband $\mathcal{P}$ and $\mathcal{S}$ with $\mathcal{P}' = \mathcal{P} - \mathcal{P}''$ and $\mathcal{S}' = \mathcal{S} - \mathcal{S}''$, where $\mathcal{P}''$ and $\mathcal{S}''$ are don't care bands in passband and stopband, respectively.) calculate the scaling factor, $\lambda^*(\boldsymbol{h}_q)$, corresponding to each set of coefficients, $\boldsymbol{h}_q$, obtained in the above steps.
4. Calculate the cost function values, $e(\boldsymbol{h}_q, \lambda^*(\boldsymbol{h}_q))$, for each $(\boldsymbol{h}_q, \lambda^*(\boldsymbol{h}_q))$. Let $e_i(\boldsymbol{h}_{q_i}, \lambda_i^*(\boldsymbol{h}_{q_i}))$ denote the i_{th} smallest value of $\{e(\boldsymbol{h}_q, \lambda^*(\boldsymbol{h}_q)); q \in \mathcal{D}\}$.
5. For each $e_i, i = 1, \cdots, m_1$, construct a subregion $\mathcal{A}_i$ as follows: For each $n \in \{1, \cdots, l\}$, let $h_i^u(n) \in I$ and $h_i^l(n) \in I$ be the least upper bound and greatest lower bound of $\{ \frac{h_0(n)}{\lambda^*(h_{q_i - m_2\Delta})} \frac{h_0(n)}{\lambda^*(h_{q_i + m_2\Delta})}\}$. Here, is a set of all integers, and m_2 is a small number (e.g. 2). We define $A_i(n) = \{h(n) \in I : h_i^l(n) \leq h(n) \leq h_i^u(n)\} \cup \{h_{q_i}(n)\}$, and $\mathcal{A}_i = \{(h(0), \cdots, h(l)) : h(n) \in A_i(n), n = 1, \cdots, l\}$. Let

   $$\mathcal{A} = \mathcal{A}_1 \cup \cdots \cup \mathcal{A}_{m_1},$$

   $\mathcal{A}$ is then a reduced search region for the power-of-two coefficients. The search over each subregion, $\mathcal{A}_i, i = 1, \cdots, m_1$, can then be carried out independently using parallel processing.

3.1.2. Solving (P_2) Over the Reduced Region $\mathcal{A}$

Problem (P_2) is a simplified version of Problem (P) with the continuous scaling factor, λ, replaced by $\lambda^*(\boldsymbol{h})$ according to Theorem (3.1). Therefore, Problem (P_2) is a pure discrete optimization problem which can be solved using a global optimization approach, and the solution of the Problem (P_2) in turn yields the solution of Problem (P).

We thus need to solve Problem (P_2) over each subregion $\mathcal{A}_i$, $i = 1, \cdots, m_1$. The simulated annealing (SA) process is an effective technique in the area of global optimization [1]. During the optimization, each iterate has to satisfy condition (1). Therefore, we propose to create the next possible iterate, $\boldsymbol{h}_{i+1}$, from current iterate, $\boldsymbol{h}_i$, using the following procedure:

- Step 1: Randomly choose a number, j, from $\{-l, \cdots, -1, 1, 2, \cdots, l\}$, and then let $h_{i+1}(|j|) = h_i(|j|) + \frac{j}{|j|}$,
- Step 2: If $\boldsymbol{h}_{i+1}$ from Step 1 satisfies the condition (1), stop. Otherwise, find k from $\{1, \cdots, j-1, j+1, \cdots, l\}$ such that the number of SPT terms of $(h_{i+1}(k) + 1)$ (or $(h_{i+1}(k) - 1)$ is less than the number of SPT terms of $h_{i+1}(k)$. Then $h_{i+1}(k) = h_{i+1}(k) + 1$ (or $h_{i+1}(k) = h_{i+1}(k) - 1$).

Remark: In case the length of a filter is not very large, the number of all candidate points in a subregion will not very large. Therefore, we can simply go through these candidate points to find the minimum PWR over the whole region.

3.2. Design of $F_a(w)$

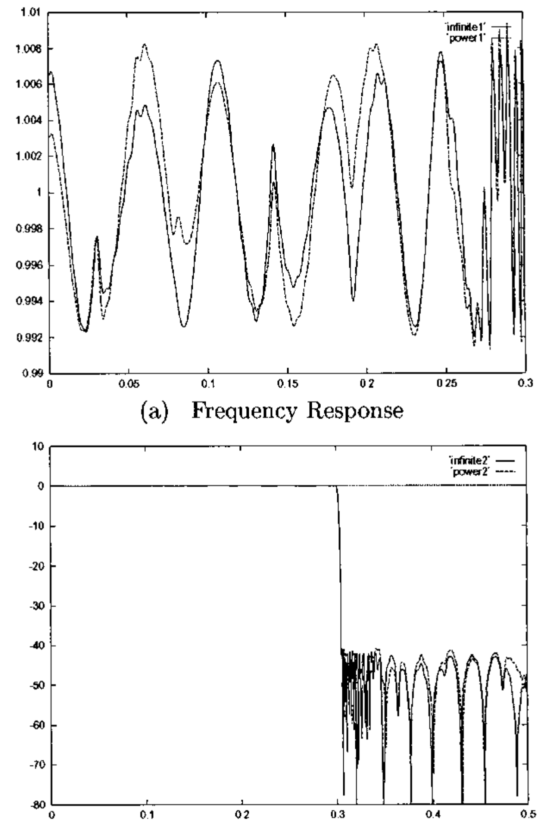
The design of F_a can be formulated as: find $\boldsymbol{h}_a$ and λ_a such that

$$e(\boldsymbol{h}_a, \lambda_a) = \max_{w \in \{\mathcal{P}_n \cup \mathcal{S}_n\}} W(w)|D(w) - \lambda_a F(w)|$$

is minimized. Here, $\boldsymbol{h}_a$ and λ_a are the power-of-two coefficients and continuous scaling factor of the filter, F_a, respectively. $W(w)$ and $D(w)$ are frequency weighting function and target objective of the system. $\mathcal{P}_n$ and $\mathcal{S}_n$ are frequencies of passbands and stopband near the transition band of $F(w)$, respectively. (For example, in the case that the passband of F_{Ma} is wider than that of F_{Mc}, $\mathcal{P}_n = (\frac{2m\pi-\theta}{M}, \frac{2m\pi+\theta}{M}]$ and $\mathcal{S}_n = [\frac{2m\pi+\phi}{M}, \frac{2(m+1)\pi-\phi}{M})$). The procedure of finding the optimal power-of-two coefficients and the optimal continuous scaling factor of the filter $F_a(w)$ is similar to the design of F_{Ma} and F_{Mc}, except that Step 3 of 3.1.1 is replaced with the use of linear programming to calculate the corresponding scaling factor $\lambda^*(\boldsymbol{h}_q)$.

4. DESIGN EXAMPLE

A linear phase low pass FIR filter with the passband frequencies $0 \leq w/2\pi \leq 0.3$ and the stopband frequencies $0.305 \leq w/2\pi \leq 0.5$ is considered. The frequency weighting function is $W(w) = 1$, that is $r = 1$. The impulse response up-sampling ratio, M, is chosen to be 9. The wordlength of the power-of-two coefficients is chosen to be 9. The estimated lengths of F_a, F_{Ma} and F_{Mc} are 47, 39 and 33, respectively, and the total numbers of the SPT terms for these subfilters are constrained by 94, 78 and 66, respectively. The parameters m_1 and m_2 in our method are chosen as 3 and 2, respectively. The minimum PWR with power-of-two coefficients, obtained using our method, is -40.56dB, which is only 0.72 dB less than the infinite precision solution of -41.27dB. The frequency response of the filters, designed by the method proposed in this paper and the infinite precision solution are depicted in the Figure 3.



(a) Frequency Response

(b) Frequency Response/dB

Fig. 3. Frequency response of the filters, obtained by the infinite precision solution, and the solution using the method proposed in this paper

5. CONCLUSION

A new method is developed for designing a FRM based FIR digital filter in the power-of-two space. Numerical results obtained from the example show that our method produced a very good quality solution with modest computational effort.

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