Digital Filters

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INTRODUCTION

A *signal* is defined as any physical quantity that varies with changes of one or more independent variables, and each can be any physical value, such as time, distance, position, temperature, or pressure (Elali, 2003; Smith, 2002). The independent variable is usually referred to as "time". Examples of signals that we frequently encounter are speech, music, picture, and video signals. If the independent variable is continuous, the signal is called *continuous-time signal* or *analog signal*, and is mathematically denoted as $x(t)$. For *discrete-time signals*, the independent variable is a discrete variable; therefore, a discrete-time signal is defined as a function of an independent variable n, where n is an integer. Consequently, $x(n)$ represents a sequence of values, some of which can be zeros, for each value of integer n. The discrete–time signal is not defined at instants between integers, and it is incorrect to say that $x(n)$ is zero at times between integers. The amplitude of both the continuous and discrete-time signals may be continuous or discrete. *Digital signals* are discrete-time signals for which the amplitude is discrete. Figure 1 illustrates the analog and the discrete-time signals.

Most signals that we encounter are generated by natural means. However, a signal can also be generated synthetically or by computer simulation (Mitra, 2006).

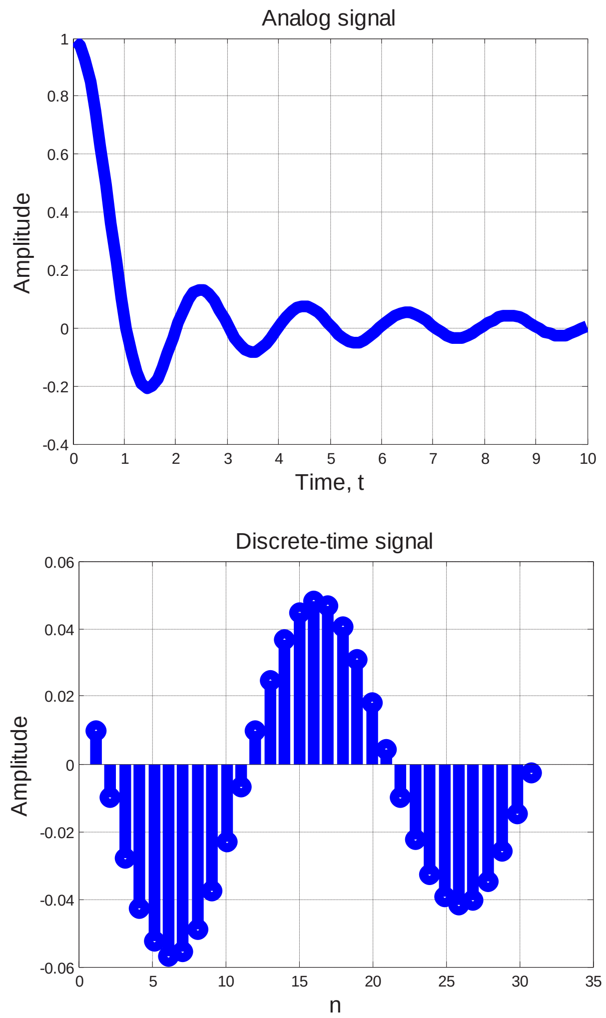
Signal carries information, and the objective of *signal processing* is to extract useful information carried by the signal. The method of information extraction depends on the type of signal and the nature of the information being carried by the signal. "Thus, roughly speaking, signal processing is concerned with the mathematical representation of the signal and algorithmic operation carried out on it to extract the information present,'' (Mitra, 2006, pp. 1).

Analog signal processing (ASP) works with the analog signals, while *digital signal processing* (DSP) works with digital signals. Since most of the signals that we encounter in nature are analog, DSP consists of these three steps:

- A/D conversion (transformation of the analog signal into the digital form);
- Processing of the digital version; and
- Conversion of the processed digital signal back into an analog form (D/A).

We now mention some of the advantages of DSP over ASP (Diniz, Silva, & Netto, 2002; Ifeachor & Jervis, 2001; Mitra, 2006; Stearns, 2002; Stein, 2000):

Figure 1. Examples of analog and discrete-time signals



- Less sensitivity to tolerances of component values and independence of temperature, aging, and many other parameters;
- Programmability, that is, the possibility to design one hardware configuration that can be programmed to perform a very wide variety of signal processing tasks simply by loading in different software;
- Several valuable signal processing techniques that cannot be performed by analog systems, such as for example linear phase filters;
- More efficient data compression (maximum amount of information transferred in the minimum amount of time);
- Any desirable accuracy can be achieved by simply increasing the word length;
- Applicability of digital processing to very low frequency signals, such as those occurring in seismic applications (An analog processor would be physically very large in size.); and
- Recent advances in very large scale integrated (VLSI) circuits make it possible to integrate highly-sophisticated and complex digital signal processing systems on a single chip.

Nonetheless, DSP has some disadvantages (Diniz, Silva, & Netto, 2002; Ifeachor & Jervis, 2001; Mitra, 2006; Stein, 2000):

- **Increased complexity:** The need for additional pre-and post-processing devices such as A/D and D/A converters and their associated filters and complex digital circuitry;
- The limited range of frequencies available for processing; and
- **Consummation of power:** Digital systems are constructed using active devices that consume electrical power, whereas a variety of analog processing algorithms can be implemented using passive circuits employing inductors, capacitors, and resistors that do not need power.

In various applications, the aforementioned advantages by far outweigh the disadvantages and, with the continuing decrease in the cost of digital processor hardware, the field of digital signal processing is developing fast. "Digital signal processing is extremely useful in many areas, like image processing, multimedia systems, communication systems, audio signal processing'' (Diniz, Silva, & Netto, 2002, pp. 2-3).

Figure 2. Digital filter

The system which performs digital signal processing, that is, transforms an input sequence $x(n)$ into a desired output sequence $y(n)$, is called a *digital filter* (see Figure 2).

We consider a filter to be a *linear-time invariant system* (LTI). The linearity means that the output of a scaled sum of the inputs is the scaled sum of the corresponding outputs, known as the principle of superposition. The time invariance says that a delay of the input signal results in the same delay of the output signal.

TIME-DOMAIN DESCRIPTION

If the input sequence $x(n)$ is a unit impulse sequence $\delta(n)$,

$$\delta(n) = \begin{cases} 1 & \text{for} \quad n = 0 \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

then the output signal represents the characteristics of the filter called the *impulse response*, and denoted by $h(n)$. We can, therefore, describe any digital filter by its impulse response $h(n)$.

Depending on the length of the impulse response $h(n)$, digital filters are divided into filters with the *finite impulse response* (FIR) and *infinite impulse response* (IIR).

In practical applications, one is only interested in designing stable digital filters, that is, whose outputs do not become infinite. The stability of a digital filter can be expressed in terms of the absolute values of its unit sample responses (Mitra, 2006; Smith, 2002),

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty. \tag{2}$$

Because the summation (2) for an FIR filter is always finite, FIR filters are always stable. Therefore, the stability problem is relevant in designing IIR filters.

The operation in time domain which relates the input signal $x(n)$, impulse response $h(n)$, and the output signal $y(n)$, is called the *convolution*, and is defined as,

$$y(n) = x(n) * h(n) = h(n) * x(n) = \sum_k h(k)x(n-k) = \sum_k x(k)h(n-k), \quad (3)$$

where * is the standard sign for convolution.

The output $y(n)$ can also be computed recursively using the following *difference equation* (Mitra 2006; Silva & Jovanovic-Dolecek, 1999),

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) + \sum_{k=1}^{N} a_k y(n-k), \quad (4)$$

where $x(n\text{-}k)$ and $y(n\text{-}k)$ are input and output sequences $x(n)$ and $y(n)$ delayed by k samples, and b_k and a_k, are constants. The order of the filter is given by the maximum value of N and M. The first sum is a *non-recursive*, while the second sum is a *recursive* part. Typically, FIR filters have only the non-recursive part, while IIR filters always have the recursive part. As a consequence, FIR and IIR filters are also known as non-recursive and recursive filters, respectively.

From (4) we see that the principal operations in a digital filter are multiplications, delays, and additions.

DIGITAL FILTERS IN THE TRANSFORM DOMAIN

The popularity of the transform domain in DSP is due to the fact that more complicated time domain operations are converted to much simpler operations in the transform domain. Moreover, different characteristics of signals and systems can be better observed in the transform domain (Mitra, 2006; Smith, 2002). The representation of digital filters in the transform domain is obtained using the *Fourier transform* and *z- transform*.

The Fourier transform of the signal $x(n)$ is defined as

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}, \quad (5)$$

where ω is digital frequency in radians and $e^{j\omega n}$ is a complex exponential sequence. In general case, the Fourier transform is a complex quantity.

The convolution operation becomes multiplication in the frequency domain,

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}), \quad (6)$$

where $Y(e^{j\omega})$, $X(e^{j\omega})$, and $H(e^{j\omega})$, are Fourier transforms of $y(n)$, $x(n)$ and $h(n)$, respectively. The quantity $H(e^{j\omega})$ is called the *frequency response* of the digital filter, and it is a complex function of the frequency ω with a period 2π. It can be expressed in terms of its real and imaginary parts, $H_R(e^{j\omega})$ and $H_I(e^{j\omega})$, or in terms of its magnitude $\left|H(e^{j\omega})\right|$ and phase $\varphi(\omega)$,

$$H(e^{j\omega}) = H_R(e^{j\omega}) + jH_I(e^{j\omega}) = \left|H(e^{j\omega})\right| e^{j\varphi(\omega)}. \quad (7)$$

The amplitude $\left|H(e^{j\omega})\right|$ is called the *magnitude response* and the phase $\varphi(\omega)$ is called the *phase response* of the digital filter. For a real impulse response digital filter, the magnitude response is a real even function of ω, while the phase response is a real odd function of ω. In some applications, the magnitude response is expressed in the logarithmic form in decibels as

$$G(\omega) = 20\log_{10}\left|H(e^{j\omega})\right| \quad \text{dB}, \quad (8)$$

where $G(\omega)$ is called the *Gain function*.

Z-transform is a generalization of the Fourier transform that allows us to use transform techniques for signals not having Fourier transform. For the sequence $x(n)$, z-transform is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad (9)$$

Z-transform of the unit sample response $h(n)$, denoted as $H(z)$, is called *system function*. Using z-transform of the Equation (4) we arrive at

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}. \quad (10)$$

For the FIR filter, all coefficients a_k, are zero, and consequently the denominator of the system function is simply 1. However, IIR filters always have the denominator different from 1.

The roots of the numerator, or the values of z for which $H(z)$=0, define the locations of the *zeros* in the complex z plane. Similarly, the roots of the denominator, or the values of z for which $H(z)$ become infinite, define the locations of the *poles*. Both poles and zeros are called *singularities*. The plot of the singularities in z-plane is called the *pole-zero pattern*. A zero is usually denoted by a circle o and the pole by a cross x. An FIR filter has only zeros (poles are in the origin), whereas an IIR filter can have either both zeros and poles, or only poles (zeros are in the origin). More detail about characteristics and applications of different FIR and IIR filters can be found in Mitra (2006), Stearns (2002), and Smith (2002).

DESIGN OF DIGITAL FILTERS

The design of digital filter is the determination of a realizable system function $H(z)$ approximating the given frequency response specification (Mitra, 2006; Smith, 2002; Stearns, 2002; White, 2000). There are two major issues that need to be answered before one can develop $H(z)$. The first issue is the development of a reasonable magnitude specification from the requirements of the filter application. The second issue is the choice on whether an FIR or an IIR digital filter is to be designed (Mitra, 2006; White, 2000).

In most practical applications, the problem of interest is the digital filter design for a given magnitude response specification. If necessary, the phase response of the designed filter can be corrected by equalizer filters (Mitra, 2006).

A filter that passes only low frequencies and rejects high frequencies is called a *lowpass filter*. The ideal lowpass filter has the magnitude specification given by

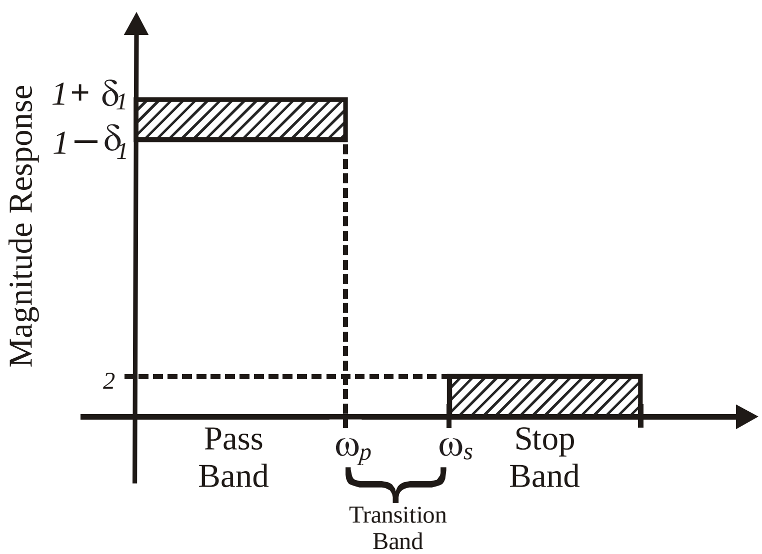
$$\left|H(e^{j\omega})\right| = \begin{cases} 1 & \textit{for} \quad |\omega| \le \omega_c \\ 0 & \textit{for} \quad \omega_c < |\omega| \le \pi, \end{cases} \qquad (11)$$

where ω_c is called *cutoff frequency*. This filter cannot be realized so the realizable specification is shown in Figure 3. The cutoff frequency ω_c is replaced by the *transition band* in which the magnitude specification is not given. The magnitude responses in the passband and the stopband are given with some acceptable tolerances, as shown in Figure 3.

The principal methods for the design of FIR filters are: *Parks McClellan algorithm*, *frequency sampling*, *window methods*, *weighted-least-squares* (WLS), and so forth. For more details, see White (2000) and Diniz, Silva, and Netto (2002).

The most widely-used methods for IIR filter design are extensions of the methods for the analog filter design (Mitra, 2006; Silva & Jovanovic-Dolecek, 1999; White, 2002). The reason is twofold. Like IIR filters, analog filters have an infinite impulse response, and

Figure 3. Lowpass filter specification



the methods for the design of analog filters are highly advanced. As a first step, the digital filter specification is converted into an analog lowpass filter specification, and an analog filter meeting this specification is designed. Next, the designed analog filter is transformed into a desired digital filter. Commonly-used transformation methods are *bilinear* and *impulse invariance method* (Mitra, 2006; Silva & Jovanovic-Dolecek, 1999). If a filter other than a lowpass filter needs to be designed, the method also includes the frequency transformation in which the designed lowpass filter is transformed into the appropriate type (highpass, bandpass, or band-rejecting filter). In this way, the Butterworth, Chebyshev I , Chebyshev II, and Elliptic digital filters can be designed. Recently, the direct IIR filter design has been proposed by Fernandez-Vazquez and Jovanovic-Dolecek (2006).

COMPARISON OF FIR AND IIR FILTERS

FIR filters are often preferred over IIR filters because they have many very desirable properties (Mitra, 2006), such as linear phase, stability, absence of limit cycle, and good quantization properties. Arbitrary frequency responses can be designed, and excellent design techniques are available for a wide class of filters.

The main disadvantage of FIR filters is that they involve a higher degree of computational complexity compared to IIR filters with equivalent magnitude response. FIR filters of length N require $(N+1)/2$ multipliers if N is odd and $N/2$ multipliers if N is even, N-1 adders, and N-1 delays. The complexity of the implementation increases with the increase in the number of multipliers.

It has been shown that for most practical filter specifications, the ratio of the FIR filter order and IIR filter order is typically of tens or more (Mitra, 2006), and as a result the IIR filter is computationally more efficient. However, if the linearity of the phase is required, the IIR filter must be equalized, and in this case the savings in computation may no longer be that significant (Mitra, 2006).

In many applications where the linearity of the phase is not required, the IIR filters are preferable because of the lower computational requirements.

OVERVIEW OF METHODS FOR FILTER DESIGN

Over the past years, there have been several attempts to reduce the number of multipliers. The Interpolated Finite Impulse Response (IFIR) filter proposed by Nuevo, Cheng, and Mitra (1984) is one of the most promising approaches for the design of low complex narrowband FIR filters. The basic idea of an IFIR structure is to implement a FIR filter as a cascade of two lower order FIR blocks, model filter, and interpolator, resulting in a less overall complexity. Different methods have been proposed to make an IFIR filter design more efficient, such as Gustafsson, Johansson, and Wanhammar (2001); Mehrnia and Willson (2004); Jovanovic-Dolecek and Mitra (2005); and Diaz-Carmona, Jovanovic-Dolecek, and Padilla (2006).

The application of the frequency-response masking technique for the design of a sharp FIR filter with a wide bandwidth was introduced in 1986 by Lim. The lower order model and masking filters are designed instead of a high-order wide-band FIR filter. Different methods have also been proposed to reduce the complexity of masking filters: Wu-Sheng and Hinamoto (2004); Saramaki and Lim (2003); Yu and Lian (2004), Jovanovic-Dolecek and Mitra (2006); Saramaki and Johansson (2001); Rodriges and Pai (2005), and so forth.

Another approach is a true multiplier-less design where the coefficients are reduced to simple integers or to simple combinations of powers of two. The main approach is based on optimizing the filter coefficient values such that the resulting filter meets the given specification with its coefficient values represented in minimum number of signed powers-of-two (MNSPT) or canonic signed digits (CSD) representations of binary digits (Bhattacharya & Saramaki, 2003; Coleman, 2002; Izydorczyk, 2006; Kotteri, Bell, & Carletta, 2003; Liu, Chen, Shin, Lin, & Jou, 2001; Vinod, Chang, & Singla, 2006; Xu, Chang, & Jong, 2006). In general, optimization techniques are complex, can require long runtimes, and provide no performance guarantees (Kotteri et al., 2003). Some authors have proposed to reduce the number of adders in the multipliers of FIR filters. The common sub-expression elimination (CSE) focus on eliminating redundant computations in multiplier blocks using the most commonly-occurring sub-expressions that exist in the CSD representation (Lin, Chen, & Jou, 2006; Maskell, Leiwo, & Patra, 2006; Vinod et al., 2006).

Another approach is based on combining simple sub-filters (Jovanovic-Dolecek, Alvarez, & Martinez, 2005; Jovanovic-Dolecek & Mitra, 2002; Tai & Lin, 1992; Yli-Kaakinen & Saramaki, 2001). In Jovanovic-Dolecek and Mitra (2002), a stepped triangular approximation of the impulse response is used which can be implemented as a cascade of a recursive running sun (RRS) filter and another RRS filter with a sparse impulse response requiring no multiplications. The efficient implementations based on rounding operation are given in Jovanovic-Dolecek and Mitra (2006).

Tai and Lin (1992) proposed a design of multiplier-free filters based on sharpening technique where the prototype filter is a cascade of the cosine filters which requires no multipliers and only some adders. However, to satisfy the desired specification, the order of the sharpening polynomial must be high, thereby resulting in high complexity.

FUTURE TRENDS

The design of FIR filters with low complexity and IIR filters with approximately linear phase are the major digital filter design tasks. In various digital signal processing applications there is a need for filter with variable frequency characteristics, for example, sampling rate conversion, echo cancellation, time-delay estimation, timing adjustment in all-digital receivers, modeling of music instruments, and speech coding and synthesis (Yli-Kaakinen & Saramaki, 2006). Variable digital filters can be constructed using either FIR or IIR filters.

Classical digital signal processing structures are the so-called single-rate systems because the sampling rates are the same at all points of the system. There are many applications where the signal of a given sampling rate needs to be converted into an equivalent signal with a different sampling rate. The main reason could be to increase efficiency or simply to match digital signals that have different rates (Jovanovic-Dolecek, 2001).

The process of converting the given rate of a signal to a different rate is called *sampling rate conversion.* Systems that employ multiple sampling rates in the processing of digital signals are called *multirate digital signal processing systems*.

Multirate digital signal processing has different applications, such as efficient filtering, sub-band coding of speech, audio and video signals, analog/digital conversion, and communications, among others. Multirate signal processing and sample rate conversion will have one of the principal tasks for signal processing of digital communications transceivers.

Software radio (SWR) is one of the key enabling technologies for the wireless revolution and is considered as one of the more important emerging technologies for the future wireless communications. Besides, instead of a traditional analog design, the software radio uses digital signal processing techniques in performing the central functions of the radio transceiver (Burachini, 2000). The application of multirate techniques to a softer radio design allows the designer to have significant latitude in selecting the system's cost, modes of operation, level of parallelism, and level of quantization noise in the system (Hentschel & Fettweis, 2002; Reed, 2002).

CONCLUSION

Digital signal processing lies at the heart of the modern technological development finding the applications in a different areas like image processing, multimedia, audio signal processing, communications, and so forth. A system which performs digital signal processing is called a digital filter. The digital filter changes the characteristics of the input digital signal in order to obtain the desired output signal. Digital filters either have a finite impulse response (FIR), or an infinite impulse response (IIR). FIR filters are often preferred because of desired characteristics, such as linear phase and no stability problems. The main disadvantage of FIR filters is that they involve a higher degree of computational complexity compared to IIR filters with equivalent magnitude response. In many applications where the linearity of the phase is not required, the IIR filters are preferable because of the lower computational requirements. Over the past years, there have been a number of attempts to reduce the complexity of FIR filters.

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KEY TERMS

Digital Filter: The digital system which performs digital signal processing, that is, transforms an input sequence into a desired output sequence.

Digital Signal: A discrete-time signal whose amplitude is also discrete. It is defined as a function of an independent, integer-valued variable *n*. Consequently, a digital signal represents a sequence of discrete values (some of which can be zeros), for each value of integer *n*.

Digital Signal Processing: Extracts useful information carried by the digital signals and is concerned with the mathematical representation of the digital signals and algorithmic operations carried out on the signal to extract the information.

FIR Filter: A digital filter with a finite impulse response. FIR filters are always stable. FIR filters have only zeros (all poles are at the origin).

IIR Filter: A digital filter with an infinite impulse response. IIR filters always have poles and are stable if all poles are inside the unit circle.

Impulse Response: The time domain characteristic of a filter and represents the output of the unit sample input sequence.

Magnitude Response: The absolute value of the Fourier transform of the unit sample response. For a real impulse response digital filter, the magnitude response is a real even function of the frequency.

Phase Response: The phase of the Fourier transform of the unit sample response. For a real impulse response digital filter, the phase response is an odd function of the frequency.

Signal: Any physical quantity that varies with changes of one or more independent variables which can be any physical value, such as time, distance, position, temperature, and pressure.

Stable Filter: A filter for which a bounded input always results in a bounded output.