Design and Applications of Digital Filters

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INTRODUCTION

Digital signal processing (DSP) is an area of engineering that "has seen explosive growth during the past three decades" (Mitra, 2005). Its rapid development is a result of significant advances in digital computer technology and integrated circuit fabrication (Jovanovic Dolecek, 2002; Smith, 2002). Diniz, da Silva, and Netto (2002) state that "the main advantages of digital systems relative to analog systems are high reliability, suitability for modifying the system's characteristics, and low cost".

The main DSP operation is digital signal filtering, that is, the change of the characteristics of an input digital signal into an output digital signal with more desirable properties. The systems that perform this task are called digital filters. The applications of digital filters include the removal of the noise or interference, passing of certain frequency components and rejection of others, shaping of the signal spectrum, and so forth (Ifeachor & Jervis, 2001; Lyons, 2004; White, 2000).

Digital filters are divided into finite impulse response (FIR) and infinite impulse response (IIR) filters. FIR digital filters are often preferred over IIR filters because of their attractive properties, such as linear phase, stability, and the absence of the limit cycle (Diniz, da Silva & Netto, 2002; Mitra, 2005). The main disadvantage of FIR filters is that they involve a higher degree of computational complexity compared to IIR filters with equivalent magnitude response (Mitra, 2005; Stein, 2000).

For example let us consider an FIR filter of length N = 11 with impulse response

$$h(n)=\begin{cases}0.8^n & for \quad 0\le n\le 10\\ 0 & otherwise\end{cases}, \tag{1}$$

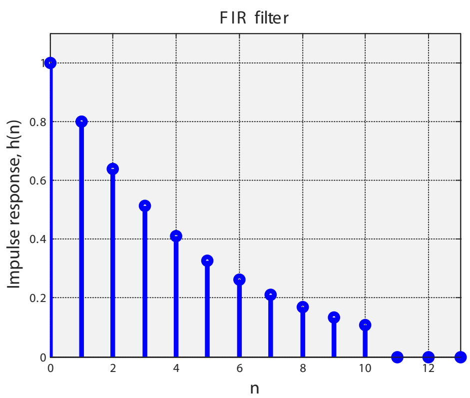
as shown in Figure 1a.

In Figure 1b the initial 20 samples of the impulse response of an IIR filter

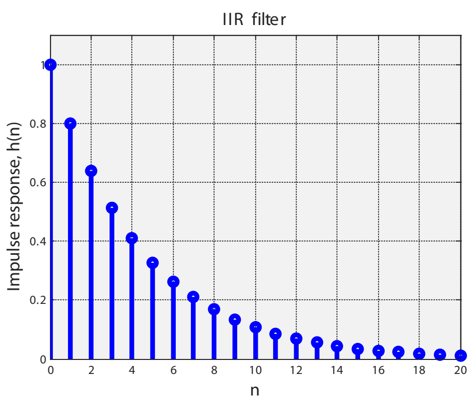
$$h(n)=\begin{cases}0.8^n & for \quad 0\le n\\ 0 & for \quad n<0\end{cases}. \tag{2}$$

are plotted.

Figure 1. Impulse responses of FIR and IIR filters



a. FIR filter



b. IIR filter

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Equation 3.

$$y(n) = x(n) * h(n) = h(n) * x(n) = \sum_k h(k)x(n-k) = \sum_k x(k)h(n-k)$$

BACKGROUND

Digital Filters in Time and Transform Domain

The operation in time domain which relates the input signal $x(n)$, impulse response $h(n)$ and the output signal $y(n)$, is called the *convolution*, and is defined in Equation 3.

The output $y(n)$ can also be computed recursively using the following *difference equation* (Mitra 2005; Proakis & Ingle, 2003),

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) + \sum_{k=1}^{N} a_k y(n-k), \quad (4)$$

where $x(n-k)$ and $y(n-k)$ are input and output sequences $x(n)$ and $y(n)$ delayed by k samples, and b_k and a_k are constants. The order of the filter is given by the maximum value of N and M. The first sum is a *nonrecursive*, while the second sum is a *recursive* part. Typically, FIR filters have only non-recursive part, while IIR filters always have the recursive part. As a consequence, FIR and IIR filters are also known as nonrecursive and recursive filters, respectively.

From (3) we see that the principal operations in a digital filter are multiplications, delays and additions. Using equation (3) we can draw the structure of the digital filter which is also known as a *Direct form* and is shown in Figure 2. More details about filter structures can be found for example in Mitra (2005).

The representation of digital filters in the transform domain is obtained using the *Fourier transform* and *z-transform*.

The Fourier transform of the signal $x(n)$ is defined as

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{j\omega n}, \quad (5)$$

where ω is digital frequency in radians and $e^{j\omega n}$ is an exponential sequence. In general case, the Fourier transform is a complex quantity.

The convolution operation becomes multiplication in the frequency domain,

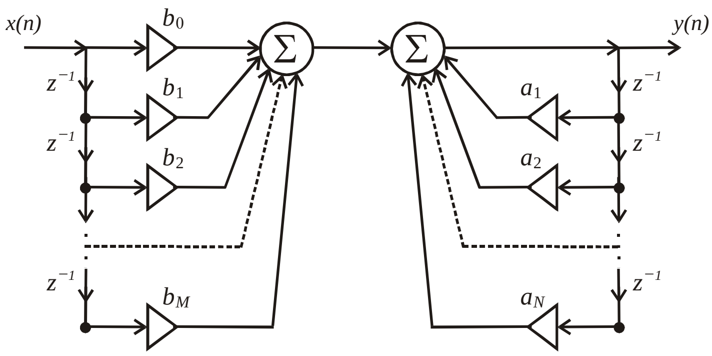
$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}), \quad (6)$$

where $Y(e^{j\omega})$, $X(e^{j\omega})$, and $H(e^{j\omega})$, are Fourier transforms of $y(n)$, $x(n)$ and $h(n)$, respectively. The quantity $H(e^{j\omega})$ is called the *frequency response* of the digital filter, and it is a complex function of the frequency ω with a period 2π. It can be expressed in terms of its real and imaginary parts, $H_R(e^{j\omega})$ and $H_I(e^{j\omega})$ or in terms of its magnitude $|H(e^{j\omega})|$ and phase $\varphi(\omega)$,

$$H(e^{j\omega}) = H_R(e^{j\omega}) + jH_I(e^{j\omega}) = |H(e^{j\omega})|e^{j\phi(\omega)}. \quad (7)$$

The amplitude $|H(e^{j\omega})|$ is called the *magnitude response* and the phase $\varphi(\omega)$ is called the *phase response* of the digital filter. For a real impulse response digital filter, the magnitude response is an even function of ω, while the phase

Figure 2. Direct form structure



response is a real odd function of ω. In some applications, the magnitude response is expressed in the logarithmic form in decibels as

$$G(\omega) = 20\log_{10}\left|H(e^{j\omega})\right| \quad \text{dB} \quad , \tag{8}$$

where $G(\omega)$ is called the *Gain function*.

For the sequence $x(n)$, z-transform is defined as

$$X(z) = \sum_{n=\infty}^{\infty} x(n) z^{-n} \ , \tag{9}$$

where z is a complex variable. All values of z for which (9) converges are called the *region of convergence* (ROC).

Z-transform of the unit sample response $h(n)$, denoted as $H(z)$, is called *system function*. Using z-transform of the Equation (4) we arrive at

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}. \tag{10}$$

The roots of the numerator, or the values of z for which $H(z)$=0, define the locations of the *zeros* in the complex z plane. Similarly, the roots of the denominator, or the values of z for which $H(z)$ become infinite, define the locations of the *poles*. Both poles and zeros are called *singularities*. The plot of the singularities in z-plane is called the *pole-zero pattern*. The zero is usually denoted by a circle (o) and the pole by a cross (x). An FIR filter has only zeros (poles are in the origin), whereas an IIR filter can have either both zeros and poles, or only poles, (zeros are in the origin). All poles of the linear-phase filter are in the origin and zeros are in ether in symmetrical positions in respect of the unit circle or on the unit circle. Its unit sample response has symmetry (Mitra, 2005; Proakis & Ingle, 2006). If the FIR filter does not have the linear phase (the unit sample response does not have symmetry) its zeros are not in symmetrical positions around the unit circle. This filter has all zeros inside the unit circle and is called a minimum-phase filter (Mitra, 2005). IIR filter which passes all frequencies without attenuation is called allpass filter. The zeros and poles of this filter are in a symmetrical positions relating to the unit circle. The filter is stable if all poles are inside the *unit circle* in z-plane. (FIR filters with the poles in origin, and IIR filters with the poles inside the unit circle.) More details about characteristics and applications of different FIR and IIR filters can be found in (Kuo, 2006; Lyons, 2004; Mitra 2005; Proakis & Ingle, 2003; Stearns, 2002; Smith, 2002, Weeks, 2006)

Transform of LP into HP Filter

Instead of designing a high-pass filter by brute force, we can transform it into a low-pass filter. We replace the desired cutoff frequencies of the high-pass filter ω_p and ω_s, by the corresponding low-pass specifications as follows:

$$\omega'_p = \pi - \omega_p$$
$$\omega'_s = \pi - \omega_s \ . \tag{11}$$

Given these specifications, a low-pass FIR filter can be designed. From this auxiliary low-pass filter, the desired high-pass filter can be computed by simply changing the sign of every other impulse response coefficient. This is compactly described as,

$$h_{HP}(n) = (-1)^n h_{LP}(n), \tag{12}$$

where $h_{HP}(n)$ and $h_{LP}(n)$ are the impulse responses of the high-pass and the low-pass filters, respectively.

Examples of Filtering

Example 1

In this example we consider a signal composed of two cosine signals $x_1(n)$ and $x_2(n)$, shown in Figures 3a, and 3b.

$$x_1(n) = \cos(0.2\pi n), x_2(n) = \cos(0.6\pi n).$$
$$x(n)=x_1(n)+x_2(n)$$

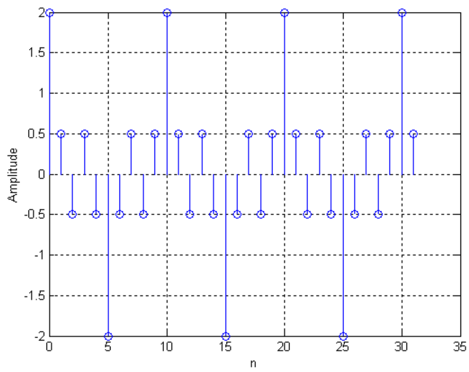
Two peaks at 0.2π and $0.6\,\pi$ in the spectral characteristic correspond to the cosine components x_1 and x_2, respectively.

Suppose we now apply low-pass (LP) filtering to the sum of these two cosine signals. The result of filtering is shown in Figures 3c and d. Notice that the second high pass cosine signal has been eliminated. To eliminate the low-pass cosine signal, we design the high-pass (HP) filter. The filtered signal is shown in Figures 3e and f.

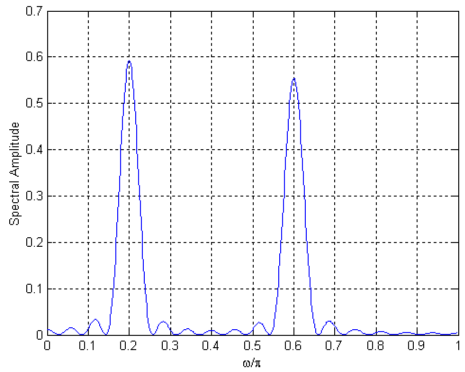
Example 2

The following figure presents an example of a speech signal (McClellan, Schafer & Yoder, 1998). We consider one part of the signal (the samples from 1,300 to 1,500), which is shown in Figures 4b and c. The low-pass filter which passes all spectral components below $0.25\,\pi$ and eliminates all spectral components higher than 0.3π (Figure 4c). The speech signal filtered by the filter is shown in Figure 4d. Notice that the resulting signal becomes smoother when higher frequencies

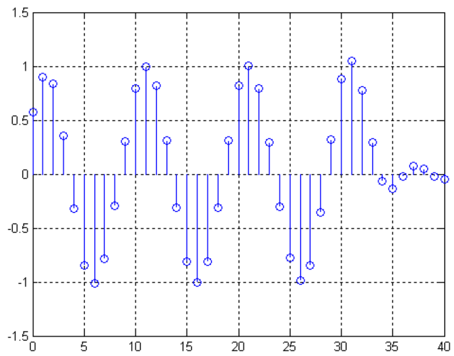
Figure 3. Sum of two cosine signals



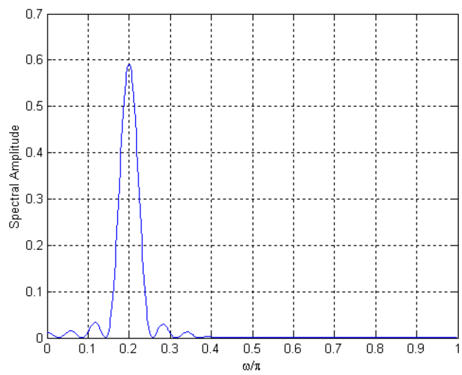
a. Time-domain



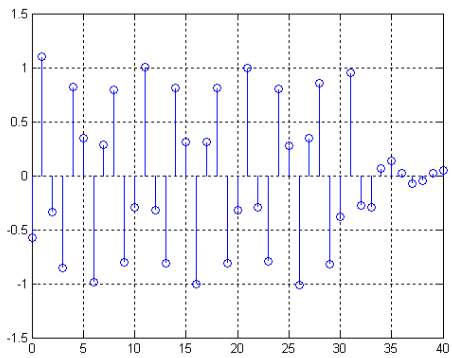
b. Frequency domain



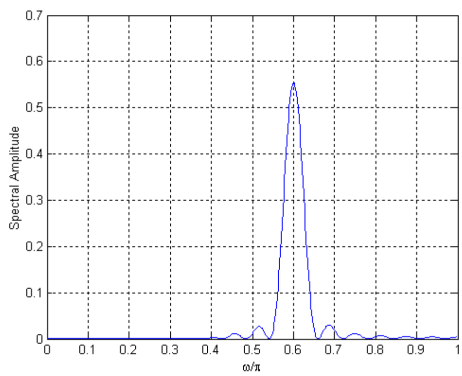
c. LP Time-domain signal



d. LP Frequency domain

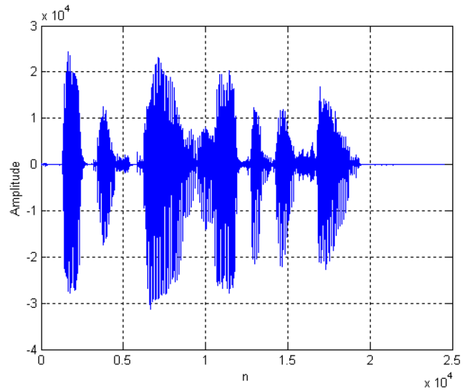


e. HP Time domain

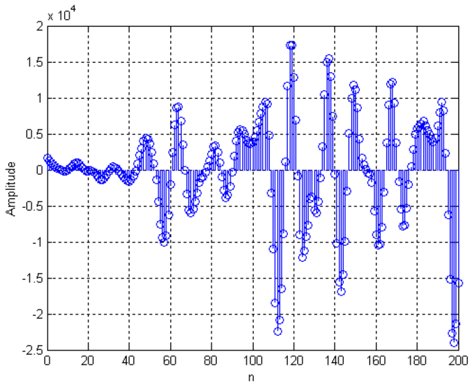


f. HP Frequency domain

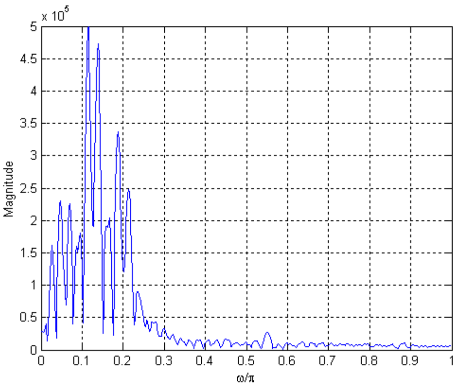
Figure 4. Sampled speech waveform



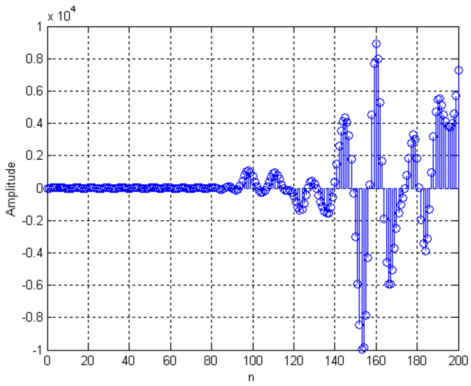
a. Speech signal



b. Part of the speech signal



c. Frequency domain



d. LP filtering

are eliminated. Therefore, low-pass filtering can be used to remove large fluctuations in the signal.

More details for digital filters for audio signal processing can be found in Meana (2007); Huang and Benesty (2004); Spanias, Painter, and Atti (2007).

Example 3

In this example we illustrate the effect of filtering of an image, generated in MATLAB, shown in Figure 5a. The noise is added to the image and the result is shown in Figure 5b. Two filters are applied to eliminate the noise. Figure 5c shows the result of applying a simple averaging filter, while Figure 5d shows the effect of applying a special filter called the median filter. Notice that the median filter is much better in removing noise.

More details about image signal processing can be found in Bose and Meyer (2006); Barnet (2007); Woods (2006); Bovik (2000).

FUTURE TRENDS

For years much effort has been made to reduce the complexities of the FIR filters (Chan, Tsui & Zhao, 2006; Izydorczyk, 2006; Lin, Chen & Jou, 2006; Macleod & Dempster, 2005; Maskell, Jussipekka & Patra, 2006; Xu, Chang & Jong, 2006). This field of research "continues to be in full of vigor as new design problems arise and innovative design techniques emerge" (Lu, 2006).

Another two direction of the research include IIR designs that are nonlinear and nonconvex and FIR filter design with certain structures that lead to nonconvex second-order or higher order design (Lu, 2006).

Another important trend is in design of variable digital filters which can be designed using either FIR or IIR filters (Yli-Kaakinen & Saramaki, 2006).

Figure 5. Removing the noise from the image

a. Image signal

b. The image with added noise

c. Filtering with averaging filter

d. Filtering with median filter

CONCLUSION

The digital filter changes the characteristics of the input digital signal in order to obtain the desired output signal. Digital filters either have a finite impulse response, (FIR), or an infinite impulse response, (IIR). FIR filters are often preferred because of desired characteristics, such as linear phase and no stability problems. The main disadvantage of FIR filters is that they involve a higher degree of computational complexity compared to IIR filters with equivalent magnitude response. In many applications where the linearity of the phase is not required, the IIR filters are preferable because of the lower computational requirements.

REFERENCES

Abeysekera, S. S. & Padhi, K. P. (2000). Design of multiplier free FIR filters using a LADF sigma-delta (Sigma-Delta) modulator. *2000 IEEE International Symposium on Circuits and Systems. Emerging Technologies for the 21st Century*. Proceedings IEEE, 2, 65-68.

Barner, K. E. (2007). *Nonlinear signal and image processing: Theory, methods, and applications*. Taylor & Francis.

Bose, T. & Meyer, F. G. (2003). *Digital signal and image processing.* New York: John Wiley & Sons.

Bovik, A. C. (2000). *Handbook of image and video processing*. Academic Press.

Chan, S. C., Tsui, K. M., & Zhaoo, S. H. (2006). A methodology for automatic syntheses of multiplier-less digital filters with prescribed output accuracy. In *Proceedings of the IEEE Conference, APCCAS* (pp. 61-64).

Coleman, J. O. (2002). Cascaded coefficient number systems lead to FIR filters of striking computational efficiency. In *Proceedings of the 2002 IEEE International Conference on Acoustics, Speech, and Signal Processing* (pp. 513-516). Piscataway, NJ.

Diniz, P. S. R., da Silva, E. A .B., & Netto, S. L. (2002). *Digital signal processing: System analysis and design*. Cambridge: Cambridge University Press.

Huang, Y. & Benestry, J. (Eds.) (2004). *Audio signal processing for next-generation multimedia communication systems.* Springer.

Ifeachor, E. C. & Jervis, B. E. (2001). *Digital signal processing: A practical approach* (2nd ed.). NJ: Prentice Hall.

Izydorczyk, J. (2006). An algorithm for optimal terms allocation for fixed point coefficients of FIR filters. In *Proceedings* *of the IEEE Conference on Circuits and Systems, ISCAS 2006* (pp. 609-611).

Jovanovic-Dolecek, G. (Ed.) (2002). *Multirate systems: Design and applications*. Hershey, PA: Idea Group Publishing.

Kuo, S. M. (2006). *Real-time digital signal processing: Implementations and applications*. Wiley.

Kuo, C. J., Chien, H. C., & Lin, W. H. (2000). Neighboring full-search algorithm for multiplierless FIR filter design. IEICE transactions on fundamentals of electronics communications & computer sciences. *Inst. Electron. Inf & Commun, 11*, 2379-2381.

Lin, M. C., Chen, H. Y., & Jou, S. J. (2006). Design techniques for high-speed multirate multistage FIR digital filters. *International Journal of Electronics, 93*(10), 699-721.

Lu, W. S. (2006). Digital filter design: Global solutions via polynomial optimization. In *Proceedings of the IEEE Conference, APCCAS* (pp. 49-52).

Lyons, R. G. (2004). *Understanding digital signal processing* (2nd ed.). Prentice Hall PTR.

Macleod, M. D. & Dempster, A. G. (2005). Multiplierless FIR filter design algorithms. *IEEE Signal Processing Letters, 12*(3), 186-189.

Meana, H. P. (Ed.) (2007). *Advances in audio and speech signal processing: Technologies and applications*. Hershey, PA: IGI Global.

Mitra, S. K. (2005). *Digital signal processing: A computer-based approach*. New York: McGraw-Hill, Inc.

Proakis, J. G. & Ingle, V. K. (2003). *A self-study guide for digital signal processing*. Prentice Hall.

Proakis, J. G. & Manolakis, D. K. (2006). *Digital signal processing* (4th ed.). Prentice Hall.

Smith, S. (2002). *Digital signal processing: A practical guide for engineers and scientists*. New York: Newnes.

Spanias, A., Painter, T., & Atti, V. (2007). *Audio signal processing and coding.* Wiley-Interscience.

Stein, J. (2000). *Digital signal processing: A computer science perspective*. New York: Wiley- Interscience.

Weeks, M. (2006). *Digital signal processing using MATLAB and wavelets*. Infinity Science Press.

White, S. (2000). *Digital signal processing: A filtering approach*. Delmar Learning.

Woods, J. W. (2006). *Multidimensional signal, image, and video processing and coding.* Academic Press.

Xu, F., Chang, C., & Jong, C. C. (2006). A new integrated approach to the design of low-complexity FIR filters. In *Proceedings of the IEEE Conference on Circuits and Systems, ISCAS 2006* (pp. 601- 604).

Yli-Kaakinen & Saramaki, T. (2001). A systematic algorithm for the design of multiplierless FIR filters. In *Proceedings of the 2001 IEEE International Symposium on Circuits and Systems* (pp. 185-188). Piscataway, NJ.

Yli-Kaakinen, & Saramaki, T. (2006). Approximately linear-phase recursive digital filters with variable magnitude characteristics. In *Proceedings of the 2006 IEEE International Symposium on Circuits and Systems* (pp. 5227-5230).

KEY TERMS

Convolution y(n)=x(n)*h(n): Time domain operation which relate the output of the digital filter y(n) with the input signal x(n) and the impulse response of the filter h(n).

Cutoff Frequencies: The frequencies which determine the passband (the frequencies which are passed without attenuation), and the stop-band (the frequencies which are highly attenuated).

Difference Equation: Time domain relation between the output and the input of digital filter in terms of coefficients which are characteristics of the filter. Generally contains recursive and nonrecursive parts.

Frequency Response $H(e^{j\omega})$: The discrete-time Fourier transform of the impulse response of the system is called the Frequency response. It provides a frequency-domain description of the system. In general, it has a complex value.

High-Pass Digital Filter: Digital filter which passes only high frequencies defined by the passband cutoff frequency and attenuates all frequencies from 0 to cutoff stopband frequency.

Impulse Response *h*(*n*): The response of a digital filter to a unit sample sequence, which consists of a single sample at index *n* = 0 with unit amplitude.

Low-Pass Digital Filter: Digital filter which passes only low frequencies defined by the passband cutoff frequency and attenuates all high frequencies from the cutoff stopband frequency to π.

Magnitude Response $\left|H(e^{j\omega})\right|$: Absolute value of the complex frequency response.

Phase Response: Phase of the complex frequency response.

Singularities: Poles and zeros of system function. Poles of system function are zeros of its denominator while zeros are zeros of its nominator.

System Function: Z-transform of the impulse response of the filter. FIR filters has only the nominator, while an IIR filter has denominator or both nominator and denominator.