Design of Digital Filters for Low Power Applications Using Integer Quadratic Programming

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Abstract. An integer quadratic programming based formulation is proposed for the design of FIR filters implemented on Digital Signal Processors (DSP). The method unifies the cost of switching activity and number of ones in coefficients and is applicable to DSPs having multiple multiply accumulate units. Four FIR filter examples are designed with the proposed method. Power simulation results show that up to 38% power reduction can be achieved in the multiply accumulate unit of a DSP using the optimized coefficients. The resulting coefficients show better performance than coefficients optimized with previously proposed methods such as reordering coefficients.

1 Introduction

Digital filters are the most frequently used elements in signal processing applications. They are realized with ASICs or can be implemented by programming of digital signal processors. They require sequential arithmetic calculations and thus consume large power and require dedicated fast hardware resources. Therefore, power aware design of digital filters is essential. In this work, we focus on FIR filters since they are the basic building block of most digital filtering structures.

FIR filtering operation of an N tap filter can be expressed by the equation

$$y[n] = \sum_{k=0}^{N-1} h[n]x[n-k] \tag{1}$$

where x represents the input data stream, h the coefficients of the filter, and y the output data stream. The low power FIR filter design problem can be divided into two categories depending on the choice of implementation: Constant coefficient and variable coefficient FIR filter synthesis.

Constant coefficient applications are also referred to as multiplierless implementation of FIR filters. Since multiplication is a combination of addition operations, the less the number of additions, the less the number of adders, which then translates to a reduction in area and power of the final implementation. The number of addition operations is determined by the number of nonzero bits in the coefficients. Many researchers have focused on this problem and proposed several optimal and suboptimal algorithms to solve the problem [1], [2], [3].

Variable coefficient implementations of FIR filters are generally realized on digital signal processors (DSP) where the filtering algorithm is translated into a series of multiply accumulate operations. The basic source of power consumption is the multiplication operation, which is performed on a dedicated multiplier unit. A filtering operation on a single multiply-accumulate (MAC) unit is shown in figure 1. The

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power dissipated in a multiplier is related to the switching activity in the multiplier, which in turn is directly affected by the switching activity (Hamming Distance) at the inputs [4].

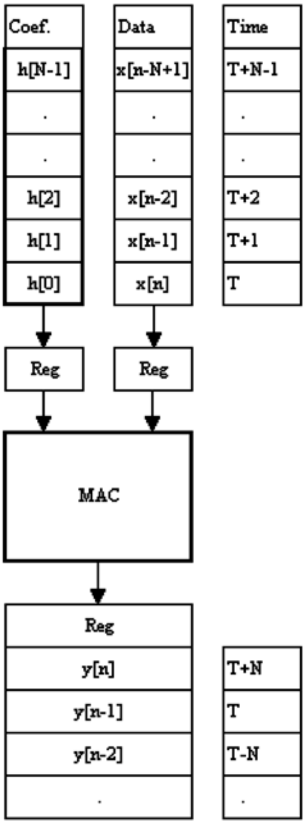

Fig. 1. FIR filtering on a single MAC unit

To reduce the power consumed in the MAC unit, the coefficients can be reordered so as to minimize the Hamming distance between successively applied coefficients [5], [6], [7]. However, reordering of coefficients requires reordering of data. One should keep in mind that data is usually correlated and thus there are very few sudden jumps between consecutive data. This then may cancel out the reduced Hamming distance for the coefficients by increasing the Hamming distance in the data stream. One can alleviate this problem by both considering the Hamming distance of the data and coefficient stream simultaneously. In this case, the possible reordering of data, especially in real-time systems or even systems where data is stored in consecutive addresses in memory, may offset expected the gains in power. Thus this approach should be restricted to problems where both data and the coefficients are readily available and reordering does not bring much power overhead.

In [4], a method that only reduces the switching activity between filter coefficients is proposed. The method formulates the coefficient optimization problem as a local search problem to find low switching activity coefficients, thus resulting in suboptimal solutions. In [9] the same problem is formulated as an integer linear programming problem targeting low Hamming distance coefficients thus reducing the power consumed. However it lacks the contribution of the number of ones in the coefficients thus resulting filters are optimum for Hamming distance but not necessarily for power.

In our work, we converted the low power FIR filter coefficient synthesis problem to a problem to find low switching activity (Hamming distance) and number of ones coefficients which is then formulated as a quadratic integer programming problem. Moreover since today's processors may possess multiple MAC units the formulation proposed handles this situation. The resulting coefficients are optimum in terms of switching activity and number of ones for the desired number of multiplier units. A couple of example filters are designed and the power performances are tested on a pre-designed multiply-accumulate (MAC) unit. The effectiveness of our approach is also shown on a processor having multiple MAC units.

2 The Formulation

The frequency response of a linear phase FIR filter having N taps is given by:

$$H(\omega) = \sum_{i=0}^{M-1} h_i T_i(\omega) \tag{2}$$

$$M = \left\lfloor \frac{N+1}{2} \right\rfloor \tag{3}$$

where $H(\omega)$ is the magnitude response without phase, T is a trigonometric function determined by the number of taps (even or odd) and the type of symmetry (symmetric or anti-symmetric) the coefficients have. The two's complement representation of a coefficient can be expressed as:

$$h_i = -x_{i,0} + \sum_{j=1}^{B-1} x_{i,j} 2^{-j} \ , \ x_{i,j} \in \{0,1\} \tag{4}$$

where B is the quantization word length, and $x_{i,j}$ corresponds to the j'th bit of coefficient h_i.

Now suppose we want to design an FIR filter having a magnitude response $H(\omega)$ for which the desired magnitude response at any frequency is given with $H_d(\omega)$, and the maximum magnitude deviation allowed at any frequency is given as $\delta(\omega)$. Then the magnitude constraints for the filter at any frequency is

$$\left|H(\omega) - H_d(\omega)\right| \le \delta(\omega), \text{ for } \ 0 \le \omega \le \pi \tag{5}$$

2.1 Formulation of the Cost of Switching Activity (Hamming Distance)

Formulation of the switching activity between successively applied coefficients is done as follows: A switching between coefficients h_i and h_{i+1}, which are quantized according to (4), at bit j occurs when Boolean XOR of the two bits $x_{i,j} \oplus x_{i+1,j}$ evaluates to a one. The arithmetic expression for the Boolean XOR operation is

$$\begin{aligned} x_{i,j} \oplus x_{i+1,j} &\equiv x_{i,j}^2 + x_{i+1,j}^2 - 2x_{i,j}x_{i+1,j} \\ &\equiv \left(x_{i,j} - x_{i+1,j}\right)^2 \end{aligned} \tag{6}$$

Having defined the cost function for the switching activity between two bits, the cost of switching from coefficient h_i to h_{i+1} is given by

$$\sum_{j=0}^{B-1} \left(x_{i,j} - x_{i+1,j}\right)^2 \tag{7}$$

where B is the coefficient wordlength. The total cost of switching of an FIR filter having N taps is

$$C_{swa} = \sum_{i=0}^{N-2} \sum_{j=0}^{B-1} \left(x_{i,j} - x_{i+1,j}\right)^2 \tag{8}$$

Since for a linear phase filter the coefficients are symmetric the above cost function reduces to

$$C_{swa} = 2\sum_{i=0}^{M-2} \sum_{j=0}^{B-1} \left(x_{i,j} - x_{i+1,j}\right)^2 \tag{9}$$

where M is calculated using (3).

When there are more than one MAC unit then coefficients are assumed to be applied in the following sequence: Assuming N number of taps and P MAC units then coefficients applied to a MAC unit are h_i, h_{i+P}, h_{i+2P}, …. An example is shown in figure 2 for four MAC units and an FIR filter having ten coefficients. The new formulation of switching activity of successively applied coefficients for P MAC units is

$$C_{swa} = \sum_{p=0}^{P-1} \sum_{i=0}^{K-1} \sum_{j=0}^{B-1} \left(x_{p+iP,j} - x_{p+(i+1)P,j}\right)^2, \quad K = \left\lfloor \frac{N-1-p}{P} \right\rfloor \tag{10}$$

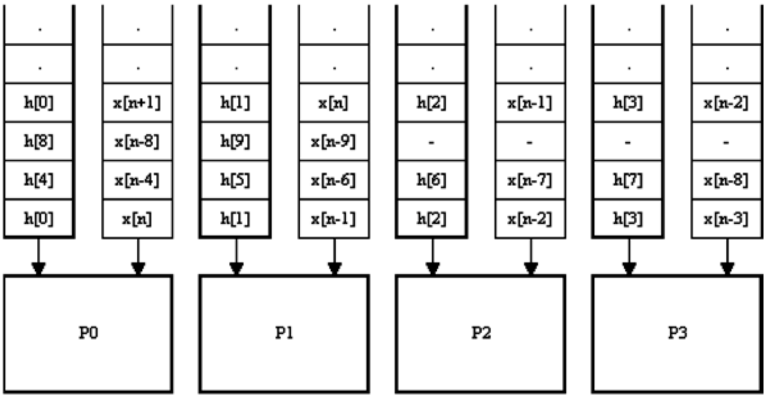

Fig. 2. FIR filtering on multiple MAC units

2.2 Formulation of the Cost of Number of Ones

Formulation of the number of ones in the coefficients of an FIR filter is straightforward. For a linear phase FIR filter having N taps and for which the coefficients are represented in two's complement representation using (4) the cost of number of ones in the coefficients can be expressed as

$$C_{one} = \sum_{i=0}^{N-1}\sum_{j=0}^{B-1} x_{i,j}\ , \quad x_{i,j} \in \{0,1\} \tag{11}$$

Since for a linear phase filter the coefficients are symmetric the above cost function reduces to

$$C_{one} = 2\sum_{i=0}^{M-1}\sum_{j=0}^{B-1} x_{i,j}\ , \quad x_{i,j} \in \{0,1\} \tag{12}$$

where M is calculated using (3).

2.3 Formulation of the Problem

Having defined the cost functions of switching activity and number of ones three sets of optimized coefficients can be obtained: minimum number of ones (MONE), minimum switching activity (MSWA) filters, and minimum switching activity and ones (MSWO) filters.

The optimization problem for MONE filters can be formulated as

$$\begin{aligned}
&\text{Minimize} && 2\sum_{i=0}^{M-1}\sum_{j=0}^{B-1} x_{i,j} \\
&\text{Subject to} && \left|H(\omega) - H_d(\omega)\right| \le \delta(\omega),\ 0 \le \omega \le \pi \\
&\text{Where} && H(\omega) = \sum_{i=0}^{M-1} h_i T_i(\omega) \\
& && h_i = -x_{i,0} + \sum_{j=1}^{B-1} x_{i,j} 2^{-j}\ ,\ \ x_{i,j} \in \{0,1\}
\end{aligned} \tag{13}$$

This problem can be solved optimally using integer programming.

The optimization problem for MSWA filters for a filter core having P MAC units can be formulated as

$$\begin{aligned}
&\text{Minimize} && \sum_{p=0}^{P-1}\sum_{i=0}^{K-1}\sum_{j=0}^{B-1}\left(x_{p+iP,j} - x_{p+(i+1)P,j}\right)^2 \\
&\text{Subject to} && \left|H(\omega) - H_d(\omega)\right| \le \delta(\omega) \quad , \quad 0 \le \omega \le \pi \\
&\text{Where} && H(\omega) = \sum_{i=0}^{M-1} h_i T_i(\omega) \\
& && h_i = -x_{i,0} + \sum_{j=1}^{B-1} x_{i,j} 2^{-j} \quad , \quad x_{i,j} \in \{0,1\} \\
& && K = \left\lfloor \frac{N-1-p}{P} \right\rfloor
\end{aligned} \tag{14}$$

Due to the quadratic term in the objective function this problem can be formulated as an integer quadratic problem. By introducing new variables this problem can be converted to an integer linear programming problem as described in [9].

By combining the switching activity cost and number of ones cost the optimization problem for MSWO filters for a filter core having P MAC units can be formulated as

$$\begin{aligned} &\text{Minimize} && \sum_{p=0}^{P-1}\sum_{i=0}^{K-1}\sum_{j=0}^{B-1}\left(x_{i,j}-x_{i+1,j}\right)^2+\sum_{i=0}^{N-1}\sum_{j=0}^{B-1}x_{i,j} \\ &\text{Subject to} && \left|H(\omega)-H_d(\omega)\right|\le\delta(\omega) \quad , \qquad 0\le\omega\le\pi \\ &\text{Where} && H(\omega)=\sum_{i=0}^{M-1}h_iT_i(\omega) \\ & && h_i=-x_{i,0}+\sum_{j=1}^{B-1}x_{i,j}2^{-j}\ ,\ x_{i,j}\in\{0,1\} \\ & && K=\left\lfloor\frac{N-1-p}{P}\right\rfloor \end{aligned} \tag{15}$$

where the cost of switching activity is given the same weight as the cost for number of ones.

3 Design Examples

In this section results for four filters taken from [4] are given. The problems are solved using ILOG CPLEX quadratic programming tool. The characteristics of the filters are shown in table 1.

Table 1. Filter characteristics

Filter	Number of Taps	Sampling Freq.	Passband		Stopband	
			Cutoff Freq.	Ripple (dB)	Cutoff Freq.	Ripple (dB)
B	24	16kHz	3kHz	0.20	4.5kHz	42
D	28	12kHz	2kHz	0.12	3kHz	45
E	34	12kHz	2.2kHz	0.16	3.1kHz	49
F	29	10kHz	2kHz	0.05	3kHz	40

Table 2 shows the results for switching activity and number of ones counts in the coefficients generated using five methods, namely NOPT, RORD, MONE, MSWA, and MSWO. NOPT coefficients are the coefficients generated using MATLAB's Remez function and quantized to 16 bits. Thus they are the reference coefficients for which no optimization is done. RORD coefficients are actually NOPT coefficients reordered by the method given in [5] for minimum switching activity. MSWA coefficients are the coefficients optimized for minimum switching activity using the formulation in (14) with ILOG CPLEX integer programming tool. MONE coefficients are the coefficients optimized for minimum number of ones using the formulation in (13) with ILOG CPLEX integer programming tool. MSWO coefficients are the coefficients optimized for both minimum number of ones and minimum switching activity using the formulation in (15) with ILOG CPLEX integer programming tool. The optimization problems were solved on a PC having INTEL P4 1.7GHz processor with 256 MB of RAM. Since solution time of integer programming problems depend on the number of variables the method proposed in [3] is used to determine the boundary values of the coefficients that satisfy the filter constraints. Using these boundary values the most significant bits of the coefficients can be determined. Hence a reduction in the number of variables is achieved.

The power performance of the generated coefficients are tested on a single MAC unit having a 16 bit Booth encoded Wallace tree multiplier and 40 bit accumulator. The MAC unit is synthesized with AMS 0.6μ technology cell library. FIR filtering is performed on 15625 samples of voice data quantized to 16 bits. Power simulations were done with an event driven gate-level simulator using a variable delay model which accounts for glitches. The operating frequency was taken to be 1MHz and supply voltage to be 5V. The resulting power dissipations are given in table 3.

The percentage power reduction is calculated by taking the NOPT coefficients' power as reference. The results indicate that by just reducing the switching activity between coefficients the best power performance cannot be achieved. By reordering coefficients one can get 19% reduction in power. Minimum switching activity (MSWA) coefficients could achieve a power reduction of 22% on average. The best power performance is obtained from MSWO coefficients having a power reduction of 35% on average. MONE coefficients have a comparable performance to MSWO coefficients with 30% power reduction on average. When design time is important, which might be the case for filters having large number of coefficients, MONE coefficients are preferable to MSWO coefficients.

Table 2. Switching activity counts and number of ones in synthesized filters for one MAC unit

Filter	Optimization method	Number of Switching	Number of Ones	Time (sec)
B	NOPT	178	186	-
	RORD	59	189	<1
	MONE	94	88	20
	MSWA	82	150	800
	MSWO	90	90	340
D	NOPT	188	244	-
	RORD	68	244	<1
	MONE	116	140	24
	MSWA	100	174	1200
	MSWO	102	144	80
E	NOPT	246	272	-
	RORD	75	272	<1
	MONE	148	140	300
	MSWA	122	290	13742
	MSWO	140	146	5328
F	NOPT	216	245	-
	RORD	59	245	<1
	MONE	122	61	4
	MSWA	108	328	2240
	MSWO	118	63	66

Another set of coefficients were generated targeting a filter core having four MAC units. The coefficients are generated for filter B. The optimization method used is MSWO but now targeting 4 MAC units, i.e. P=4 in (15). The resulting coefficients' switching activity counts for each MAC unit are given in table 4. The switching activity counts are compared to those coefficients generated using methods NOPT, and MSWO targeting one MAC unit.

The power performances of the coefficients are tested using the same MAC unit mentioned above. The operating frequency is 1MHz and supply voltage 5V. The resulting average power dissipation in each MAC unit is given in table 5. The performance of the coefficients generated by the method MSWO targeting 4 units is the best, as expected. However there is a little performance increase (3%) over MSWO coefficients targeting one MAC unit.

Table 3. Power simulation results using one MAC unit

Filter	Optimization	Power (uW)	Reduction (%)
B	NOPT	1317	-
	RORD	1077	18.2
	MONE	890	32.4
	MSWA	982	25.4
	MSWO	850	35.4
D	NOPT	1324	-
	RORD	1083	18.2
	MONE	913	31.0
	MSWA	957	27.7
	MSWO	874	34.0
E	NOPT	1321	-
	RORD	1065	19.4
	MONE	978	26.0
	MSWA	1137	13.9
	MSWO	899	31.9
F	NOPT	1249	-
	RORD	993	20.5
	MONE	787	37.0
	MSWA	1001	19.9
	MSWO	775	38.0

Table 4. Switching activity counts and number of ones in synthesized filters for four MAC units

Filter	Optimization method	Number of Switching				Number of Ones
		MAC 0	MAC 1	MAC 2	MAC 3	
B	NOPT	47	38	37	46	186
	MSWO for 1 MAC	36	22	24	36	90
	MSWO for 4 MAC	22	16	16	22	92

Table 5. Power simulation results using four MAC units

Filter	Optimization method	Power (uW)				Average Reduction (%)
		MAC 0	MAC 1	MAC 2	MAC 3	
B	NOPT	1523	1357	1341	1525	-
	MSWO for 1 MAC	994	937	906	981	33.4
	MSWO for 4 MAC	944	875	855	972	36.5

4 Conclusion

In this paper we have demonstrated the formulation of finding power optimum coefficients for realization of FIR on programmable DSPs. The coefficient optimization results for 4 low pass FIR filters were shown. The results indicate that when minimization of switching activity is our goal the most effective method is to reorder coefficients. However when it comes to power performance coefficients optimized with our method, which minimizes the number of ones in addition to switching activity, outperformed reordered coefficients in all cases. The effectiveness of the formulation on DSPs having multiple units is also shown on a design example.

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