FRM Approach for Hilbert Transformer Synthesis

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***Abstract*—Sharp Hilbert transformer has high complexity resulting in high power consumption, low throughput rate, and high implementation cost. The frequency-response masking (FRM) technique has been used very successfully for reducing the complexity of sharp filters including half-band filters by large factors. There is a very close relationship between a half-band filter and a Hilbert transformer. In this paper, we present the synthesis of very sharp Hilbert transformer using the FRM technique via the synthesis of a very sharp half-band filter. The resulting Hilbert transformer not only has a higher throughput rate but also has lower power consumption, implementation cost, and physical size when compared to the conventional minimax optimum design.**

***Index Terms*—Frequency-response masking. FIR filter. FRM. Hilbert transformer. Sparse coefficient filter. Low power design.**

I. Introduction

The FRM (frequency-response masking) technique [1-53] has been used successfully for the synthesis of sharp filters producing designs with very high computational efficiency. The consequence is a significant reduction in power consumption, implementation cost, and physical size and increase in throughput rate among other benefits. The FRM technique has been extended to the synthesis of various types of filters [21-38]. Implementations on various platforms [39-41] have also been investigated. The FRM technique has also been used in a great variety of applications [42-53].

The synthesis of half-band filter using the FRM technique has been reported [21-23]. There is a very close relationship between a half-band filter and a Hilbert transformer. In this paper, we present the synthesis of very sharp Hilbert transformer using the FRM technique via the synthesis of a very sharp half-band filter. The resulting Hilbert transformer not only has a higher throughput rate but also has lower power consumption and implementation cost as well as a smaller physical size when compared to the conventional minimax optimum design. In order to simplify notation, we shall ignore causality (which can be restored easily by introducing appropriate delays) by assuming that all filters are zero-phase filters. In the classical FRM technique [1], the z-transform transfer function, $H(z)$ of a filter is synthesized as a system of sub-filters as in (1).

$$H(z) = H_{Ma}(z)H_a(z^M) + (1-H_a(z^M))H_{Mc}(z) \quad (1)$$

In (1), $H_a(z^M)$ is the z-transform transfer function of a filter derived by replacing each delay of a prototype band edge shaping filter by M delays. The functions $H_{Ma}(z)$ and $H_{Mc}(z)$ are the z-transform transfer functions of the masking filters. The synthesis structure for (1) is shown in Fig. 1.

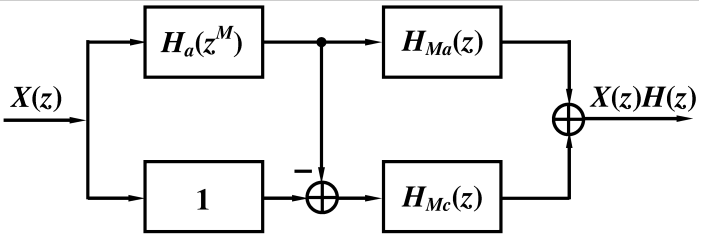

Fig. 1 Structure of a filter synthesized using the FRM technique. The transfer function "1" may be replaced by $z^{-\bullet}$ to restore causality.

II. Half-band filter synthesized using the FRM technique

In [21], a method for the synthesis of half-band filter using the frequency-response masking technique was proposed. For the purpose of defining the notations used in this paper, the essential points in [21] are summarized as follows. Let $H_a(z)$ be the z-transform transfer function of a prototype unity-gain half-band band edge shaping filter given by

$$H_a(z) = ½ + A(z) \quad (2)$$

Let the length of $H_{Ma}(z)$ be $2K + 1$. Thus, $H_{Ma}(z)$ may be written as

$$H_{Ma}(z) = h_{Ma}(0) + \sum_{k=1}^{K} h_{Ma}(k)\left(z^k + z^{-k}\right) \quad (3)$$

Define the z-transform transfer function $B(z)$ and $C(z)$ as in (4) and (5), respectively.

$$B(z) = h_{Ma}(1)(z + z^{-1}) + h_{Ma}(3)(z^3 + z^{-3}) + \ldots \quad (4)$$

$$C(z) = h_{Ma}(0) + h_{Ma}(2)(z^2 + z^{-2}) + h_{Ma}(4)(z^4 + z^{-4}) + \ldots \quad (5)$$

Substituting (4) and (5) into (3), we have

$$H_{Ma}(z) = B(z) + C(z) \quad (6)$$

In [21], $H_{Mc}(z)$ is given by

$$H_{Mc}(z) = 1 - H_{Ma}(-z) \quad (7)$$

From (6) and (7), and note that $B(-z) = -B(z)$ and that $C(-z) = C(z)$, we have

$$H_{Mc}(z) = 1 + B(z) - C(z) \quad (8)$$

Substituting (2), (6), and (8) into (1), we have,

$$H(z) = ½ + B(z) + A(z^M)(2C(z) - 1) \quad (9)$$

The half-band transfer function $H(z)$ synthesized using (9) is shown in Fig. 2 where M is odd.

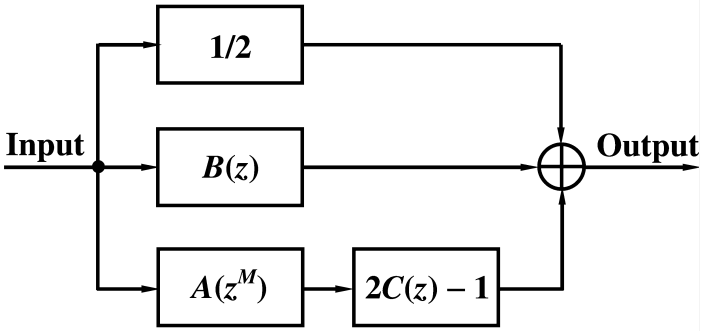

Fig. 2 FRM structure for half-band filter.

III. Deriving the Hilbert Transformer from the Half-Band Filter

A Hilbert transformer may be derived from a unity-gain half-band filter by subtracting the constant ½ from its transfer function and then modulating the remaining coefficients by $e^{j n\pi/2}$; the effect of this action on the frequency response is shown in Fig. 3. This will produce an odd length Hilbert transformer with a passband gain of 0.5. For an odd length Hilbert transformer, every other of its impulse response is zero; an even length Hilbert transformer may be derived by dropping these zero coefficients.

The transfer function of the Hilbert transformer, $H_H(z)$ is given by

$$H_H(z) = 2B(jz) + 2A(j^M z^M)(2C(jz) - 1) \quad (10)$$

The implementation structure is shown in Fig. 4.

In (10), the transfer functions $A(j^M z^M)$, $B(jz)$, and $2C((jz) - 1)$ may be written as

$$A(j^M z^M) = j \times A_0(z) \quad (11a)$$

$$B(jz) = j \times B_0(z) \quad (11b)$$

$$2C(jz) - 1 = j \times C_0(z) \quad (11c)$$

where $A_0(z)$, $B_0(z)$, and $C_0(z)$ are zero phase z-transform transfer functions. Thus, $H_H(z)$ may be written as

$$H_H(z) = j \times H_{H0}(z) \quad (12)$$

where $H_{H0}(z)$ is a zero phase z-transform transfer function. The factor j in (12) represents the 90° phase shift of the Hilbert transformer. The factor j arises because a zero phase $H(z)$ (see equation (1)) is used in the derivation. The factor j does not exist if a causal half-band filter is used as the prototype filter. Consider using the causal half-band filter with linear phase z-transform transfer function $z^{-\text{odd}}H(z)$ as the prototype filter. After replacing z by jz, the z-transform transfer function of the Hilbert transformer will become $(jz)^{-\text{odd}}H_H(z) = j \times z^{-\text{odd}}$ $(2B(jz) + 2A(j^M z^M)(2C(jz) - 1))$ and will not have the factor j; the required 90° phase shift is inherent in the transfer function.

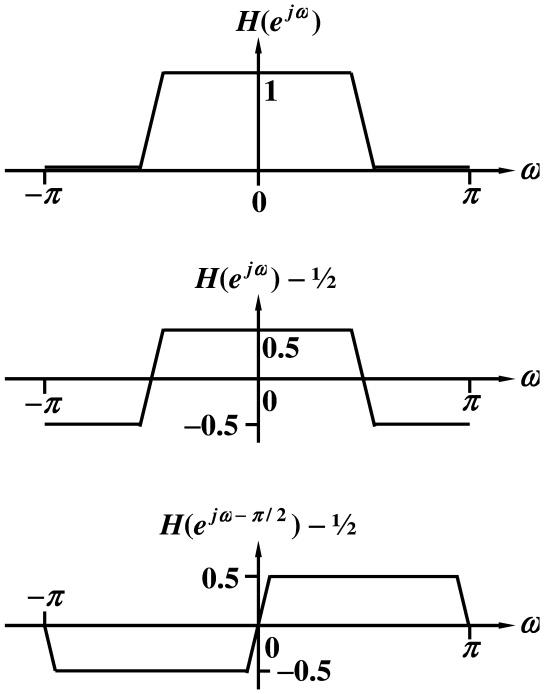

Fig. 3 Frequency response of Hilbert transformer derived from half-band filter.

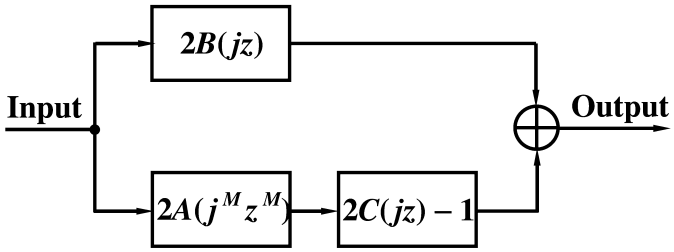

Fig. 4 Structure of Hilbert transformer implemented using the FRM technique.

IV. An Example

In a particular system, the application required a Hilbert transformer with the following specifications: sampling rate = 32 kHz, lower band edge = 100 Hz, peak ripple magnitude = 0.0001. These requirements correspond to a Hilbert transformer with transition width 0.003125 f_s where f_s is the sampling frequency. The estimated length of a direct form Chebyshev optimum design meeting the specification was 803.

Using the FRM technique, the prototype half-band filter has transition width 0.00625 f_s. The coefficient values for a design with $M = 7$ are shown in Tables I and II. The frequency response of the resulting Hilbert transformer is shown in Fig. 5. The FRM technique reduces the number of multipliers by a factor of 3 when compared to the Chebyshev optimum design.

TABLE I COEFFICIENT VALUES OF $H_a(z)$.

$h_a(-57) = 0.00000885 = h_a(57)$	$h_a(-27) = -0.00056200 = h_a(27)$
$h_a(-55) = -0.00000449 = h_a(55)$	$h_a(-25) = 0.00068783 = h_a(25)$
$h_a(-53) = 0.00001756 = h_a(53)$	$h_a(-23) = -0.00083008 = h_a(23)$
$h_a(-51) = -0.00001766 = h_a(51)$	$h_a(-21) = 0.00100667 = h_a(21)$
$h_a(-49) = 0.00003458 = h_a(49)$	$h_a(-19) = -0.00121242 = h_a(19)$
$h_a(-47) = -0.00004162 = h_a(47)$	$h_a(-17) = 0.00146971 = h_a(17)$
$h_a(-45) = 0.00006479 = h_a(45)$	$h_a(-15) = -0.00178378 = h_a(15)$
$h_a(-43) = -0.00008143 = h_a(43)$	$h_a(-13) = 0.00219478 = h_a(13)$
$h_a(-41) = 0.00011413 = h_a(41)$	$h_a(-11) = -0.00275833 = h_a(11)$
$h_a(-39) = -0.00014371 = h_a(39)$	$h_a(-9) = 0.00369024 = h_a(9)$
$h_a(-37) = 0.00019008 = h_a(37)$	$h_a(-7) = -0.05641928 = h_a(7)$
$h_a(-35) = -0.00023681 = h_a(35)$	$h_a(-5) = 0.01421025 = h_a(5)$
$h_a(-33) = 0.00030199 = h_a(33)$	$h_a(-3) = -0.05641928 = h_a(3)$
$h_a(-31) = -0.00037143 = h_a(31)$	$h_a(-1) = 0.29595677 = h_a(1)$
$h_a(-29) = 0.00046212 = h_a(29)$	$h_a(0) = 0.50000000$

TABLE II COEFFICIENT VALUES OF $H_{Ma}(z)$.

$h_{Ma}(-34) = -0.00695946 = h_{Ma}(34)$	$h_{Ma}(-16) = -0.09554157 = h_{Ma}(16)$
$h_{Ma}(-33) = -0.00001780 = h_{Ma}(33)$	$h_{Ma}(-15) = 0.29831766 = h_{Ma}(15)$
$h_{Ma}(-32) = 0.01728009 = h_{Ma}(32)$	$h_{Ma}(-14) = 0.11868809 = h_{Ma}(14)$
$h_{Ma}(-31) = 0.00012423 = h_{Ma}(31)$	$h_{Ma}(-13) = -0.27023419 = h_{Ma}(13)$
$h_{Ma}(-30) = -0.02805677 = h_{Ma}(30)$	$h_{Ma}(-12) = -0.23909232 = h_{Ma}(12)$
$h_{Ma}(-29) = -0.00018857 = h_{Ma}(29)$	$h_{Ma}(-11) = 0.23203805 = h_{Ma}(11)$
$h_{Ma}(-28) = 0.03361170 = h_{Ma}(28)$	$h_{Ma}(-10) = 0.30992696 = h_{Ma}(10)$
$h_{Ma}(-27) = 0.02809509 = h_{Ma}(27)$	$h_{Ma}(-9) = -0.18133683 = h_{Ma}(9)$
$h_{Ma}(-26) = -0.05102311 = h_{Ma}(26)$	$h_{Ma}(-8) = -0.39206726 = h_{Ma}(8)$
$h_{Ma}(-25) = -0.06916630 = h_{Ma}(25)$	$h_{Ma}(-7) = 0.19651044 = h_{M}(7)$
$h_{Ma}(-24) = 0.06194209 = h_{Ma}(24)$	$h_{Ma}(-6) = 0.39309235 = h_{Ma}(6)$
$h_{Ma}(-23) = 0.11227940 = h_{Ma}(23)$	$h_{Ma}(-5) = -0.05509426 = h_{Ma}(5)$
$h_{Ma}(-22) = -0.07434883 = h_{Ma}(22)$	$h_{Ma}(-4) = -0.41030147 = h_{Ma}(4)$
$h_{Ma}(-21) = -0.13452774 = h_{Ma}(21)$	$h_{Ma}(-3) = -0.03690918 = h_{Ma}(3)$
$h_{Ma}(-20) = 0.03216557 = h_{Ma}(20)$	$h_{Ma}(-2) = 0.41307057 = h_{Ma}(2)$
$h_{Ma}(-19) = 0.20409112 = h_{Ma}(19)$	$h_{Ma}(-1) = 0.31313419 = h_{Ma}(1)$
$h_{Ma}(-18) = 0.02892611 = h_{Ma}(18)$	$h_{Ma}(0) = 0.00000000$
$h_{Ma}(-17) = -0.24842371 = h_{Ma}(17)$	

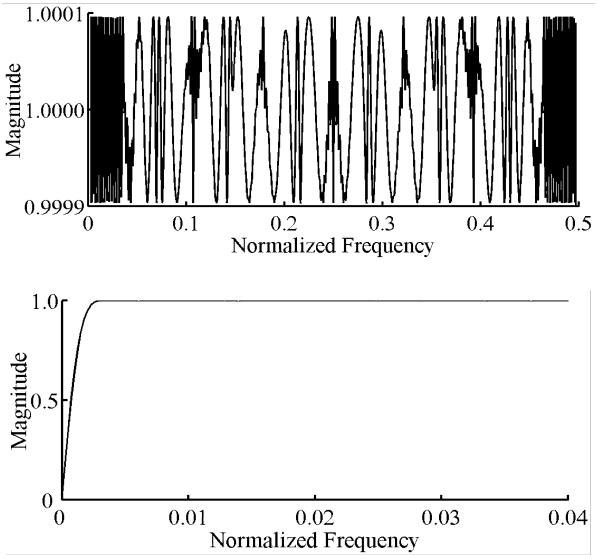

Fig. 5 Frequency response plots for the Hilbert transformer.

V. CONCLUSION

An FRM Hilbert transformer synthesis technique is presented. The technique produces significant reduction in hardware complexity resulting in design with lower power consumption and implementation cost as well as higher throughput rate.

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