Interpolation Factor Analysis for Jointly Optimized Frequency-Response Masking Filters

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***Abstract*— The introduction of nonlinear optimization techniques to the frequency response masking (FRM) filter has changed the way on how an FRM filter being synthesized. It allows all subfilters in an FRM structure to be optimized at the same time resulting further savings in the number of multipliers. In this paper, we present a set of design equations that helps to determine filter lengths and interpolation factor in an FRM filter under joint optimization approach. It is shown by means of examples that the proposed design equations lead to a better estimation of the optimum interpolation factor.**

I. Introduction

Frequency response masking (FRM) approach is one of the most computationally efficient techniques for the design of linear phase arbitrary passband sharp FIR digital filters [1]. An FRM filter contains three subfilters: a bandedge shaping filter $H_a(z)$ and two masking filters $H_{Ma}(z)$ and $H_{Mc}(z)$ as shown in Figure 1. The interpolated bandedge shaping filter $H_a(z^M)$ plays two roles in an FRM filter, it creates a narrow transition band for the overall filter and forms an arbitrary passband with the help of its complementary filter $H_c(z^M)$. The drawback is that the interpolation introduces extra passbands that fall into the stopband of the overall filter. Two masking filters are cascaded to the outputs of $H_a(z^M)$ and its complement, respectively, to remove the unwanted passbands. The principle of an FRM filter is best illustrated by its frequency responses of various subfilters as shown in Figure 2. More details of FRM can be found in [1], [2].

In an FRM filter the overall complexity is depended on the interpolation factor M. A large M reduces the complexity of $H_a(z)$ at a cost of longer masking filters. There exists an optimum interpolation factor for a given set of specifications that leads to a minimum number of multipliers in an FRM filter. Such an optimum M can be estimated using a formula given in [2]:

$$M \approx \frac{1}{2\sqrt{\Delta F}} \tag{1}$$

where ΔF is the normalized transition bandwidth of an FRM filter. The derivation of (1) is based on an assumption that FRM filters reach minimum complexity when the transition bandwidths of two masking filters are the same. Such an assumption does not hold for the most of designs if all subfilters are designed separately using an iterative design procedure [2]. Nevertheless, (1) reduces the search space for an optimum M to a small set of integers and a simple program can be developed to find bandedges of all subfilters.

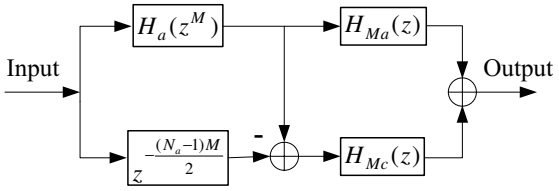

Fig. 1. A basic realization structure for the FRM filter

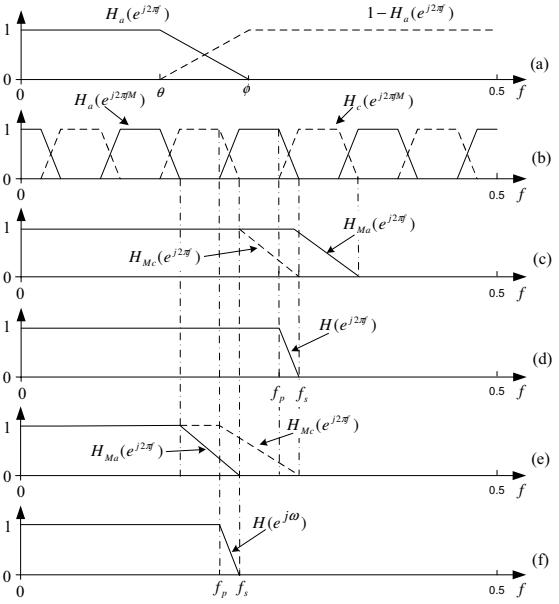

Fig. 2. The frequency responses of various filters in the FRM approach

Much research effort has been invested to improve the computational efficiency of FRM filters in recent years. A notable effort is to employ nonlinear optimization techniques to jointly optimize 3 subfilters in an FRM structure [3]–[8]. Such an approach leads to additional savings around 20% in terms of number of multipliers. However, it was reported in [3]

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that (1) is no longer accurate for a jointly optimized design. The authors of [3] suggested a new formula to estimate the optimum M, i.e.

$$M = \frac{1}{\sqrt{3.2\Delta F}} \tag{2}$$

Design examples shown that (2) gives a better estimation than that of (1). But the estimated value is still a short distance away from the optimum M and requires a few iterations to reach an optimal design. In this paper we present an improved estimation for the optimum M based on a few hundred FRM filters designed by sequential quadratic programming (SQP) algorithm [9]. The proposed formula produces much more accurate M than that of (1) or (2).

The rest of paper is organized as follows. In Section II, we highlight the differences between the original and jointly optimized FRM filters and introduce a new formula to estimate the length of the bandedge sharping filter. In Section III, further observations are made to masking filters and an equation is derived to estimate the lengths of two masking filters. A new expression for the estimation of the optimum M is presented in Section IV and a conclusion is drawn in Section V.

II. Filter Length Estimation for Bandedge Shaping Filter

It was noted that the length of bandedge shaping filter $H_a(z)$ does not change much under the joint optimization approaches compared with traditional FRM designs. For most of FRM designs, the lengths of bandedge shaping filters are very close to those designed by linear programming in [1]. However, the joint optimization alters the frequency response of the bandedge shaping filter greatly as shown in Figures 3(a) and (b). The frequency response changes from a typical lowpass function to a square waveform alike. Such a change has two major impacts on the FRM approach. First, the optimum interpolation factor moves away from the value in an original FRM approach. Second, the transition bands of two masking filters are widened and they become closer to each other. As a result, the analysis done in [2] are no longer valid. In order to develop a new expression for optimum M, we look at bandedge shaping filter and masking filters separately. In reference [1], the length of bandedge shaping filter is estimated by assuming that the ripple magnitude of $H_a(z)$ are 85% of the allowed ripple magnitude. Use such an assumption, we have the following equation according to the filter length estimation formula [10].

$$N_a(\Delta F, \delta_p, \delta_s) = \frac{-20\log_{10}\sqrt{\delta_p\delta_s} - 11.59}{14.6M\Delta F} + 1 \tag{3}$$

Equation (3) can be further refined to

$$N_a(\Delta F, \delta_p, \delta_s) = \frac{-8.38\log_{10}(\delta_p\delta_s) - 5.41}{14.1M\Delta F} + 1.13 \tag{4}$$

The above equation was the result of a few hundred FRM lowpass filter designs based on SQP optimization and provides a good estimation for the length of a bandedge shaping filter. However, such an expression does not give us a full picture

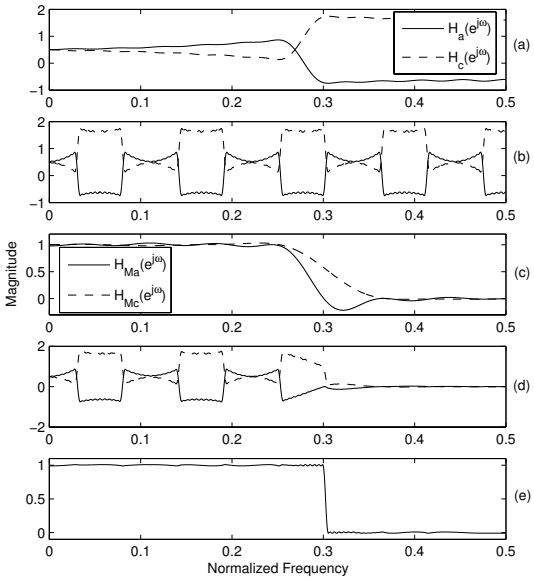

Fig. 3. The frequency responses of various filters in jointly optimized FRM approach

of the interpolation factor. We need to find the filter lengths of two masking filters before embarking on the optimum M.

III. Masking Filter Length Estimation

It is easy to verify from Figures 3(c) and (d) that the transition bandedges of two masking filters are very difficult to determine. They depend heavily on the bandedge shaping filter. To show this, let us consider the design of a lowpass filter with the following specifications: normalized passband and stopband edges are at 0.191 and 0.197, respectively. Passband and stopband ripples are both 0.01. The optimum interpolater factor is 9 after exhaustive search. The lengths of 3 subfilters are $N_a = 39$, $N_{Ma} = 27$, and $N_{Mc} = 19$, where N_a is for bandedge shaping filter, N_{Ma} and N_{Mc} are for two masking filter. Its frequency response is shown in Figure 4. However, for the same M value we can find another set of subfilters which satisfy the given specifications while have the same number of taps as in above. The filter lengths are $N_a = 39$, $N_{Ma} = 19$, and $N_{Mc} = 27$ and the frequency response is shown in Figure 5. This example tells us that it is difficult to estimate the filter length of each masking filter due to their dependence on bandedge shaping filter. We noticed although the length of each masking filter varies for a given M but the sum of N_{Ma} and N_{Mc} are kept to a near constant value. Therefore, we sort to find the sum of N_{Ma} and N_{Mc}, instead of trying to estimate the length of each masking filter.

To search minimal N_{Ma} and N_{Mc}, we start the FRM filter design with initial values of N_{Ma} and N_{Mc} which are determined as in [1] and check the frequency response of the resulting overall filter. If it meets the given specifications, we

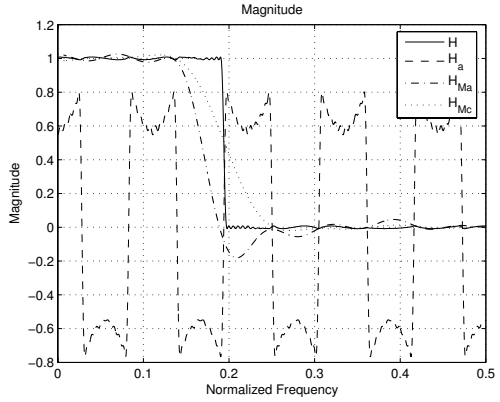

Fig. 4. Magnitude Frequency Responses of an FRM filter with $N_a = 39$, $N_{Ma} = 27$, and $N_{Mc} = 19$

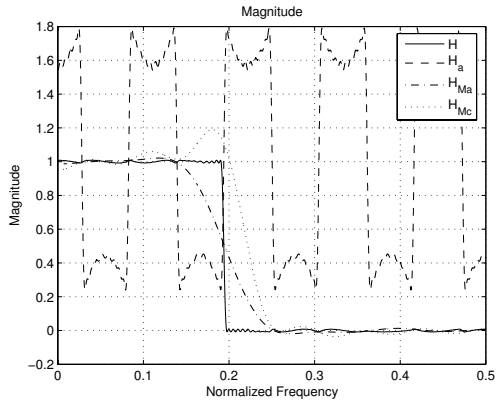

Fig. 5. Magnitude Frequency Responses of an FRM filter with $N_a = 39$, $N_{Ma} = 19$, and $N_{Mc} = 27$

reduce both N_{Ma} and N_{Mc} by one and design all subfilters again. Repeat this process until no reduction can be made to masking filters. At this point, one of N_{Ma} and N_{Mc} might be shortened further. Continue to change N_{Ma} or N_{Mc} until neither of N_{Ma} and N_{Mc} can be reduced any more. Therefore, minimal N_{Ma} and N_{Mc} are found.

Let us define $N_{msum} = N_{Ma} + N_{Mc}$. Obviously, N_{msum} will be affected by four parameters: passband ripple δ_p, stopband ripple δ_s, the normalized transition bandwidth ΔF of an FRM filter, and the interpolation factor M of bandedge shaping filter $H_a(z)$. To determine the relationship between N_{msum} and 4 parameters, we carry out a few experiments. It is easy to verify that N_{msum} has a linear relationship with the logarithm of passband and stopband ripples, i.e.

$$N_{msum} = k_1 \log_{10} \delta_p + c_1 \tag{5}$$

$$N_{msum} = k_2 \log_{10} \delta_s + c_2 \tag{6}$$

where k_1, k_2, c_1, and c_2 are constants yet to be decided.

To see how the transition bandwidth affects N_{msum}, let us fix the passband edge, ripples, and M as shown in Table I and vary the stopband edge. The filters are designed using SQP. Figure 6 shows that relationship between N_{msum} and the normalized transition bandwidth of FRM filters. It can been seen that N_{msum} varies a little for different transition bandwidths.

TABLE I
FILTER SPECIFICATIONS FOR FIGURE 6

Legend	f_p	δ_p	δ_s	M
data1	0.14	0.03	0.008	9
data2	0.18	3.16×10^{-2}	3.16×10^{-3}	7
data3	0.24	3.16×10^{-2}	0.001	11
data4	0.34	0.01	0.001	15

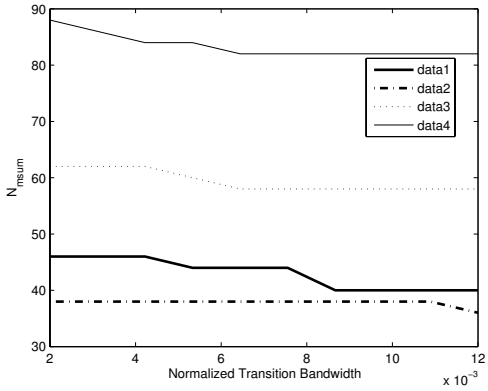

Fig. 6. Relationship between N_{msum} and transition bandwidth of the overall FRM filter

To identify the relationship between N_{msum} and M, we choose a set of lowpass filters with different bandedges and passband and stopband ripples as shown in Table II. For each set of specifications, we design the FRM filters using SQP by varying M from 4 to 19. The sums of masking filter lengths are calculated and shown in Figure 7. It is not difficult to conclude from Figure 7 that the sum of masking filter lengths can be approximated by a linear relationship of M, i.e.

$$N_{msum} = k_3 M + c_3 \tag{7}$$

where k_3 and c_3 are constants yet to be found.

Combining equations (5), (6), and (7), we get a new expression for N_{msum}

$$N_{msum} = kM \log_{10}(\delta_p \delta_s) + c \tag{8}$$

The values of k and c in equation (8) is determined through a few hundreds of FRM design examples with different specifications. Finally, we have

$$N_{msum} = -1.145M \log_{10}(\delta_p \delta_s) + 5.58 \tag{9}$$

TABLE II

SPECIFICATIONS FOR FIGURE 7

Legend	f_p	f_s	δ_p	δ_s
data1	0.2	0.206	0.001	0.01
data2	0.3	0.3035	0.01	0.01
data3	0.36	0.364	3.16×10^{-2}	3.16×10^{-3}

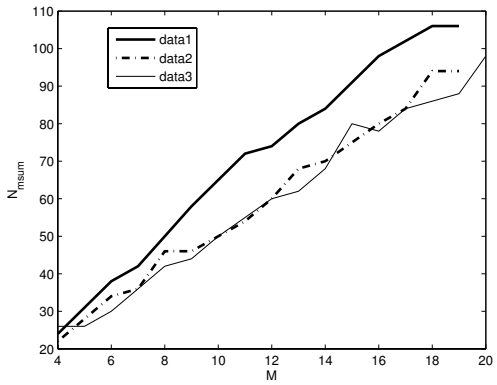

Fig. 7. Relationship between the sum of masking filter lengths and interpolation factor M

IV. OPTIMUM INTERPOLATION FACTOR

According to equations (4) and (9), the complexity of an FRM filter can be estimated by

$$\begin{aligned} N=&N_a + N_{msum} \\ =&\frac{-8.38\log_{10}(\delta_p\delta_s) - 5.41}{14.1M\Delta F} \\ &-1.145M\log_{10}(\delta_p\delta_s) + 6.71 \end{aligned} \tag{10}$$

An optimum M that minimizes the N can be easily found from equation above.

$$M_{opt} = \sqrt{\frac{8.38\log_{10}(\delta_p\delta_s) + 5.41}{16.14\Delta F\log_{10}(\delta_p\delta_s)}} \tag{11}$$

Generally speaking, $8.38\log_{10}(\delta_p\delta_s)$ is much larger than 5.41. If we ignore 5.41 and make a little compensation to it, the estimation of M_{opt} becomes

$$M_{opt} \approx \frac{1}{\sqrt{2.25\Delta F}} \tag{12}$$

The results of equation (11) and (12) are for an FRM filter jointly optimized by a nonlinear optimization method. Meanwhile, equation (12) is a function of ΔF, the transition bandwidth of the overall filter. This is consistent with equations (1) and (2).

To compare the accuracy of different estimations, two design examples are chosen for verification. The first example is taken from [1] with following parameters: $\delta_p = 0.01, \delta_s = 0.01, f_p = 0.3$, and $f_s = 0.305$. The second example is taken from [3] with following parameters: $\delta_p = 0.01, \delta_s = 0.001, f_p = 0.2$, and $f_s = 0.201$. Table III lists results calculated according to different equations and actual optimal values of M. From Table III, it is clear that equation (11) and (12) give very good estimation of M when compared with equations (1) and (2).

TABLE III

ACCURACY COMPARISON

Equation	Example in [1]	Example [3]
(1)	7.07	15.81
(2)	7.91	17.67
(11)	9.33	21.26
(12)	9.43	21.08
Actual Optimal M	9	21

V. CONCLUSION

In this paper, we have presented new formulas to estimate the length of bandedge shaping filter and the sum of length two masking filters in an FRM approach, which is designed by sequential quadratic programming (SQP). Base on these formulas, a new expression is derived to estimate the optimum value of M, the interpolation factor of bandedge shaping filter. The new expressions are suitable for FRM filters designed by other nonlinear optimization techniques.

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