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19 FOURIER SERIES OPTIMIZATION: MINIMIZING THE NUMBER OF NONZERO COEFFICIENTS

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Abstract: The frequency response of a finite impulse response (FIR) digital filter may be expressed in the form of a Fourier serious. The coefficients of the Fourier serious have a one-to-one correspondence relationship to the impulse response of the filter. Specifically, the number of Fourier series coefficients in the Fourier series corresponds to the number of multipliers required in the implementation of the FIR filter. Thus, minimizing the number of nontrivial Fourier series coefficient required to approximate a given function is important.

In this paper, we present the frequency response masking technique for developing a Fourier series approximation to a desired function with the minimum number of nontrivial coefficients. The number of coefficients required is only a small fraction of that required by the optimum Chebyshev approximation.

Key words: Fourier series; frequency response masking; digital filter.

1 INTRODUCTION

There is an electronic device called finite impulse response digit filter or, in short, FIR filter. It is frequency selective; it passes signals within a frequency band and rejects signals outside the selected frequency band. Figure 1.1 shows the structure of an FIR filter.

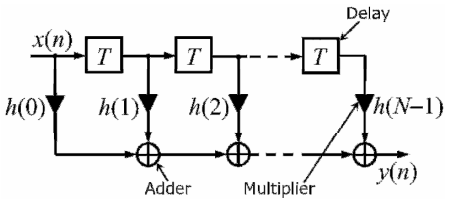

Figure 1.1 Structure of an FIR filter

Filters have very wide applications. For example, when you tune in to a radio station, the filter in your radio set passes only the station of your choice and rejects all other transmissions. In a world without filter, when you turn on your radio receiver, you will be listening to all broadcasts simultaneously! The three fundamental building blocks of a digital filter are (a) multiplier, (b) adder, and (c) delay. Among these three fundamental building blocks, the multiplier consumes the most power and is the most complex component. Reducing the number of multipliers will reduce power consumption and lower manufacturing cost. The n^{th} impulse response of the filter shown in Figure 1.1 is $h(n)$. The n^{th} impulse response is also called the n^{th} coefficient of the filter. For a linear phase FIR filter, the impulse response may be symmetrical or anti-symmetrical. If it is symmetrical, then $h(0) = h(N-1), h(1) = h(N-2), \cdots$, and $h(n) = h(N-1-n)$. If it is anti-symmetrical, then $h(0) = -h(N-1), h(1) = -h(N-2), \cdots$, and $h(n) = -h(N-1-n)$. The general form of the frequency response for an FIR filter is

$$H(\omega) = \sum_{n=0}^{N-1} h(n) e^{jn\omega} \tag{1.1}$$

Specific form of the frequency response for symmetrical or anti-symmetrical impulse response filter depends on whether N is odd or even and whether the impulse response is symmetrical or anti-symmetrical. If $N$is odd and the impulse response is symmetrical, its frequency response, $H(\omega)$, is given by Rabiner and Gold (1975)

$$H(\omega) = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-1}{2}} h\left(\frac{N-1}{2} - n\right) cos(n\omega) \tag{1.2a}$$

If N is even and the impulse response is symmetrical, its frequency response, $H(\omega)$, is given by Rabiner and Gold (1975)

$$H(\omega) = 2\sum_{n=0}^{\frac{N}{2}} h\left(\frac{N-1}{2} - n\right) cos((n-0.5)\omega) \tag{1.2b}$$

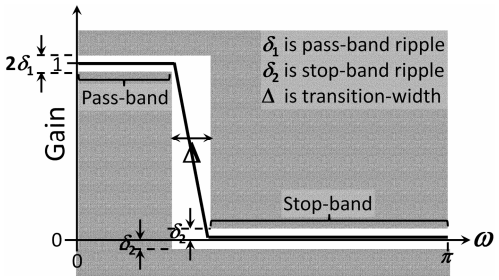

Figure 1.2 Typical frequency response requirement of a filter

If N is odd and the impulse response is anti-symmetrical, its frequency response, $H(\omega)$, is given by Rabiner and Gold (1975)

$$H(\omega) = h\left(\frac{N-1}{2}\right) + 2\sum_{i=0}^{\frac{N-1}{2}} h\left(\frac{N-1}{2} - n\right) e^{jn\omega} \tag{1.2c}$$

If N is even and the impulse response is anti-symmetrical, its frequency response, $H(\omega)$, is given by Rabiner and Gold (1975)

$$H(\omega) = h\left(\frac{N-1}{2}\right) + 2\sum_{i=0}^{\frac{N-1}{2}} h\left(\frac{N-1}{2} - n\right) e^{jn\omega} \tag{1.2d}$$

(1.2a) through (1.2d) are Fourier series. To design a digital filter is to find a Fourier series for $H(\omega)$ such that $H(\omega)$ lies within a permitted region as shown in Figure 1.2. In Figure 1.2, the permitted region is the white region within the plot area. The band where $H(\omega)$ is$\approx$ 1 is called the passband. The band where $H(\omega)$ is ≈ 0 is called the stopband. The transition band is the band between passband and stopband. The width of the transition band is called the transition width. It is well known that, for a given passband and stopand ripple requirement, the filter length (N) is inversely proportional to its transition width (Δ). Figure 1.3 shows a filter length vs transition width plot for a series of lowpass filters with peak-to-peak passband ripple $2\delta_1 = 0.0233$ (0.2dB) and stopband ripple $\delta_2 = 0.01$ (-40dB). As a result of this inverse proportion relationship between filter length and transition width, sharp filters are very complex. The complexity of a filter is measured by the number of multipliers required in its implementation; the more is the required number of multipliers, the more complex is the filter. The sharpness of a filter is measured by its transition width; the narrower is the transition width, the sharper is the filter.

When two filters with frequency responses $H_1(\omega)$ and $H_2(\omega)$, respectively, are connected in series as shown in Figure 1.4(a), the resultant frequency response $H(\omega) = H_1(\omega)\ H_2(\omega)$. When two filters with frequency responses $H_2(\omega)$and$H_2(\omega)$, respectively, are connected in parallel as shown in Figure 1.4(b), the resultant frequency response $H(\omega) = H_1(\omega) + H_2(\omega)$.

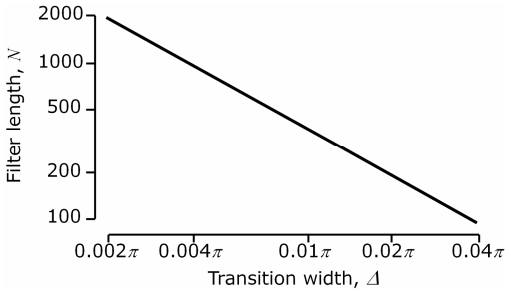

Figure 1.3 Filter length vs transition width plot for a lowpass filter with passband ripple2 δ_1=0.0233 (0.2dB) peak-to-peak and stopband ripple δ_2= 0.01 (-40dB) peak

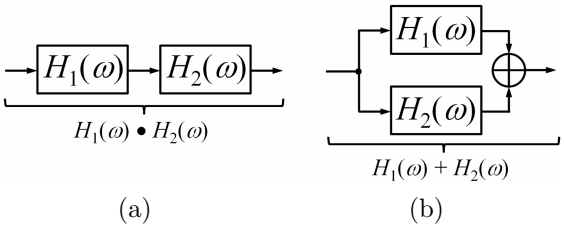

Figure 1.4 Cascade and parallel connections of filters

2 THE FREQUENCY RESPONSE MASKING (FRM) APPROACH

The design of an FIR digital filter is a process of finding a Fourier series or a set of Fourier series, which when added and/or multiplied among them, approximates a given function within a predefined tolerance. Consider a function $H_a(\omega)$ as shown in Figure 2.1.

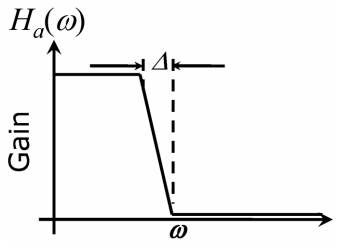

Figure 2.1 Plot of $H_a(\omega)$

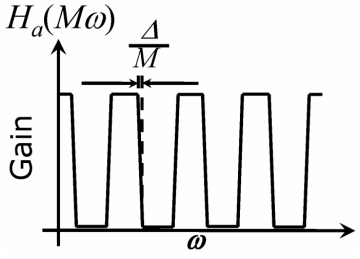

Figure 2.2 Plot of $\hat{H}(\omega)$

Let

$$\hat{H} = H_a(M\omega) \tag{2.1}$$

$\hat{H}(\omega)$is plotted in Figure 2.2. It can be seen from Figure 2.2 that the transition width for$\hat{H}(\omega)$isΔ/M, which is a factor of M smaller than that for $H_a(\omega)$.

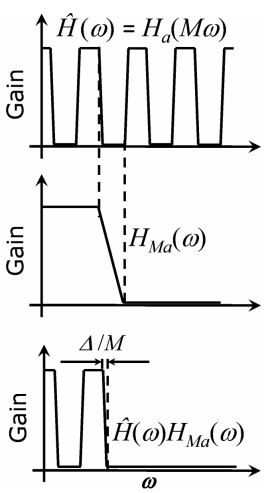

Figure 2.3 Plot of $\hat{H}(\omega)$, $H_{Ma}(\omega)$, and $\hat{H}(\omega)H_{Ma}(\omega)$

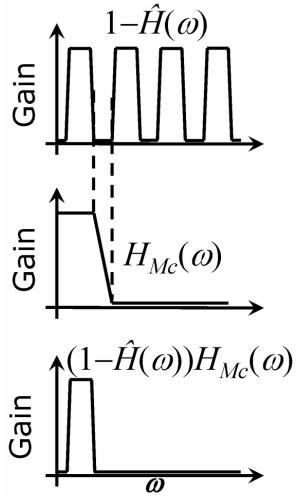

Figure 2.4 Plot of $1 - \hat{H}(\omega)$, $H_{Mc}(\omega)$, and $(1 - \hat{H}(\omega))H_{Mc}(\omega)$

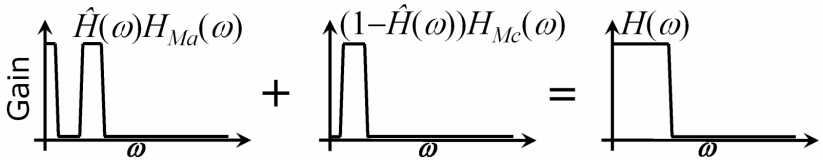

Figure 2.5 Plot of $H(\omega) = \hat{H}(\omega)H_{Ma}(\omega) + (1 - \hat{H}(\omega))H_{Mc}(\omega)$

Consider another function $H_{Ma}(\omega)$ as shown in Figure 2.3(b). The product $\hat{H}(\omega)H_{Ma}(\omega)$is plotted in Figure 2.3(c). Figure 2.2 is reproduced as Figure 2.3(a) to help visualizing the result. The transition width for$\hat{H}(\omega)H_{Ma}(\omega)$is the same as that for $\hat{H}(\omega)$. However, the frequency response shown in Figure 2.3(c) is not what is desired. For example, the desired frequency response for a lowpass filter usually resembles that shown in Figure 1.2. Comparing Figure 1.2 and Figure 2.3(c), we note that Figure 2.3(c) has an undesirable gap in its passband. This undesirable gap can be removed by adding $(1-\hat{H}(\omega))H_{Mc}(\omega)$ to $\hat{H}(\omega)H_{Ma}(\omega)$ where $H_{Mc}(\omega)$ is shown in Figure 2.4(b). Let

$$H(\omega) = \hat{H}(\omega)H_{Ma}(\omega) + (1 - \hat{H}(\omega))H_{Mc}(\omega) \tag{2.2}$$

$H(\omega)$is plotted in Figure 2.5.

From Figure 2.5, it can be seen that $H(\omega)$ has very sharp transition band (i.e. very narrow transition width). Since, its transition width is a factor of M narrower than that for $H_a(\omega)$, it will requires M time as many Fourier series

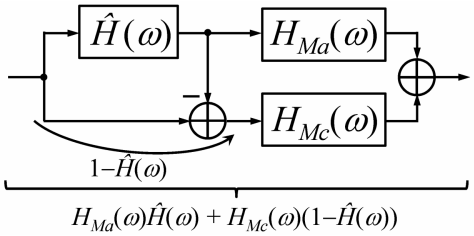

Figure 2.6 The structure of a filter synthesized using the FRM approach

terms as that for $H_a(\omega)$ if it is expressed directly in Fourier series form instead of in a sum of products of Fourier series as in (2.2).

The Fourier series representation for $\hat{H}(\omega)$ has the same number of non-zero terms as that for $H_a(\omega)$ although its transition width is a factor of M small than that for $H_a(\omega)$; this is because $\hat{H}(\omega)$ is obtained from $H_a(\omega)$ by replacing by M . Since, the number of Fourier series terms required is inversely proportional to the transition width, the number of Fourier series terms for $H_{Ma}(\omega)$ and $H_{Mc}(\omega)$ are not many; they are roughly the same as that for $H_a(\omega)$. Since none of $\hat{H}(\omega)$, $H_{Ma}(\omega)$ and $H_{Mc}(\omega)$ has a large number of Fourier series terms, the total number of Fourier series terms is small.

The above approach is called the FRM approach for designing FIR digital filter. In the approach, we obtain $\hat{H}(\omega)$ from $H_a(\omega)$ by replacing by M . In hardware implementation, replacing in $H_a(\omega)$ by M is achieved by replacing each delay in the filter implementing $H_a(\omega)$ by M delays. The structure of the filer is shown in Figure 2.6.

3 EXAMPLE

We shall illustrate the application of the FRM approach using the design of the following lowpass FIR digital filter as an example.

Passband edge : 0.3 fs. ($f_s = 2\pi$)

Stopband edge : $0.305 f_s$

Maximum passband deviation : 0.1 dB (0.0116)

Minimum stopband attenuation : 40 dB (diviation = 0.01)

The above specifications can be met using a 192 terms direct form Fourier series. Using the FRM approach with $M = 9$, a design where the number of non-zero Fourier series terms or $\hat{H}(\omega)$, $H_{Ma}(\omega)$ and $H_{Mc}(\omega)$ are 23, 21, and 17, respectively was obtained. See Figure 3.1. The total number of non-zero Fourier series terms was 23 + 21 + 17 = 61; this is significantly less than 192. The frequency response plot (i.e.$H(\omega)$ vs ω plot) is shown in Figure 3.2.

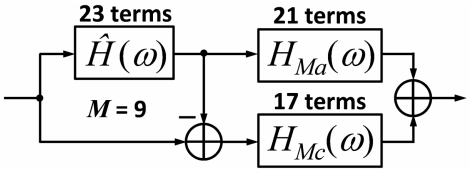

Figure 3.1 An example

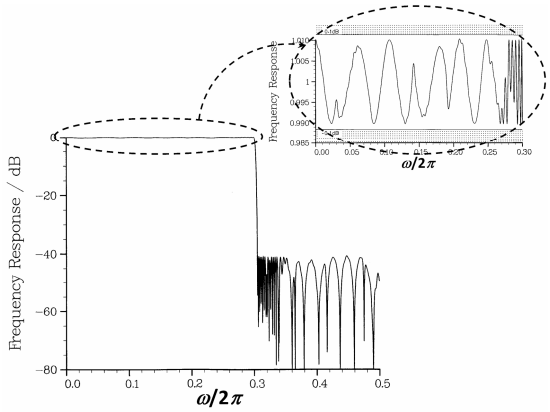

Figure 3.2 TFrequency response plots for a filter designed using the FRM approach

4 OPTIMUM M

The total number of Fourier series terms required depends on the value of M. In this section, we shall derive an expression for the optimum M. Figure 4.1 shows the relationship between the bandedges of $\hat{H}(\omega)$, $H_{Ma}(\omega)$ and $H_{Mc}(\omega)$. As can be seen from Figure 4.1, the transition width for $H_{Ma}(\omega)$ is $(2\pi - \phi - \theta)/M$ and that for $H_{Mc}(\omega)$ is $(\phi + \theta)/M$. Thus the sum of the transition widths for $H_{Ma}(\omega)$ and $H_{Mc}(\omega)$ is $2\pi/M$. Suppose the transition width of $H_{Ma}(\omega)$ is γ ; the transition width of $H_{Mc}(\omega)$ will be $2\pi/M - \gamma$. Let $L_M a$ and $L_M c$ be the number of Fourier series terms for $H_{Ma}(\omega)$ and $H_{Mc}(\omega)$, respectively. Since the number of Fourier series terms $\propto$is [transition width]$^{-1}$, we have

$$L_{Ma} + L - Mc = 1/\gamma + 1/((2\pi/M) - \gamma) \tag{4.1}$$

Differentiating $L_M a + L_M c$ with respect to γ and equate the derivative to zero, we have

$$\gamma = \pi/M \tag{4.2}$$

The transition width of $H_{Mc}(\omega)$ is $2\pi/M - \gamma = \pi/M$. Thus, $L_M a + L_M c$ is minimum when the transition width of $H_{Ma}(\omega)$ is equal to that of $H_{Mc}(\omega)$, i.e.

$$L_M a = L_M c \tag{4.3}$$

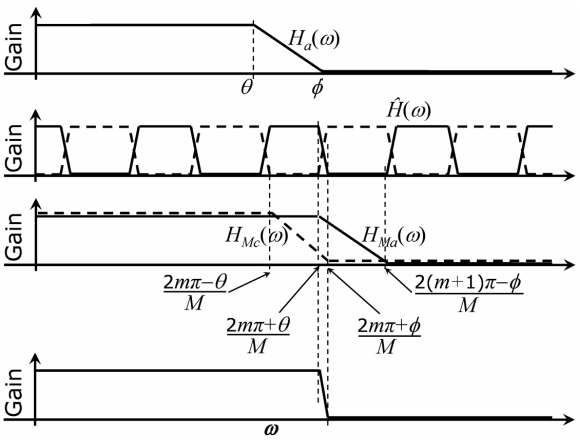

Figure 4.1 Relationship between the bandedges of $\hat{H}(\omega)$, $H_{Ma}(\omega)$ and $H_{Mc}(\omega)$

Let the transition width of the very sharp target filter be $2\pi\beta$. Let the number of Fourier series terms of the Chebyshev approximation (with transition width $2\pi\beta$) be L_0. Let the number of non-zero Fourier series terms of $H_a(\omega)$ be L_a.

Since the number of Fourier series terms is [transition width]$^{-1}$, we have

$$L_a = L_0/M \tag{4.4}$$

Since the transition width of $H_{Ma}(\omega)$ is π/M, we have

$$L_{Ma} = L_0(2\pi\beta(\pi/M)) = 2\pi\beta L_0 \tag{4.5}$$

We have shown that for the joint complexity of $H_{Ma}(\omega)$ and $H_{Mc}(\omega)$ to be minimum, $L_M a = L_M c$. Thus,

$$L_{Ma} = L_{Mc} = 2M\beta L_0 \tag{4.6}$$

The total number of Fourier series terms L is given by

$$L = L_a + L_{Ma} + L_{Mc} = (1/M + 4M\beta)L_0 \tag{4.7}$$

Differentiating $L = (1/M + 4M\beta)L_0$ with respect to M and equating the derivative to zero, and let the optimum value of M that minimizes L be M_{opt}, we have

$$M_{opt} = 1/(2\sqrt{\beta} \tag{4.8}$$

Let L_{min} be the value of L corresponding to $M = M_{opt}$.

$$L_{min} = (1/M_{opt} + 4M_{opt}\beta)L_0 = 4L_0\sqrt{\beta} \tag{4.9}$$

5 HIGHER LEVEL FRM

We have replaced $H(\omega)$ by $\hat{H}(\omega)H_{Ma}(\omega) + (1 - \hat{H}(\omega))H_{Mc}(\omega)$ where $\hat{H}(\omega) = H_a(M)$. We shall add superscripts and subscripts to the variables and rewrite the equation as

$$H(\omega) = H_a^1(M_1\omega)H_{Ma}^0(\omega) + (1 - H_a^1(M_1\omega))H_a^1(\omega) \tag{5.1}$$

If the transition width of $H(\omega)$ is very very narrow, the transition width of $H_a^1(\omega)$ may still be very narrow and so may still have many Fourier series terms. To reduce the number of Fourier series terms in $H_a^1(\omega)$, $H_a^1(\omega)$ may be written as

$$H_a^1(\omega) = H_a^1(M_2M_1\omega)H_{Ma}^1(\omega) + (1 - H_a^2(M_2M_1\omega))H_{Ma}^1(M_1\omega) \tag{5.2}$$

In fact, we can write

$$H_a^k\left(\omega\prod_{i=0}^{k} M_i\right) = H_a^{k+1}\left(\omega\prod_{i=0}^{k+1} M_i\right) H_{Ma}^k\left(\omega\prod_{i=0}^{k} M_i\right) + \left(1 - H_a^{k+1}\left(\omega\prod_{i=0}^{k+1} M_i\right)\right) + H_{Mc}^k\left(\omega\prod_{i=0}^{k} M_i\right) \tag{5.3}$$

For k = 0, 1, 2, 3, 4, $\cdots$ for as large as necessary. Note: $M_0 = 1$. Let L_{Ma}^i and L_{Mc}^i be the number of Fourier series terms in $L_{Ma}^i(\omega)$ and $L_{Mc}^i(\omega)$, respectively. We have shown that L_{Ma}= L_{Mc}= $2M\beta L_0$. Thus

$$L_{Ma}^0 + L_{Mc}^0 = 4M_1\beta L_0 \tag{5.4a}$$

$$L_{Ma}^1 + L_{Mc}^1 = 4M_2\beta L_0 \tag{5.4b}$$

$$L_{Ma}^2 + L_{Mc}^2 = 4M_3\beta L_0 \tag{5.4c}$$

$$L_{Ma}^{i-1} + L_{Mc}^{i-1} = 4M_i\beta L_0 \tag{5.4d}$$

For a K-stage recursion, the total number of Fourier series terms for $H_{Ma}^{k-1}(\omega)$, $H_{Mc}^{k-1}(\omega), \cdots, H_{Ma}^1(\omega), H_{Mc}^1(\omega), H_{Ma}^0(\omega), H_{Mc}^0(\omega)$ is

$$4M_k\beta L_0 + 4M_{k-1}\beta L_0 + 4M_1\beta L_0 = 4\beta L_0\sum_{i=1}^{K} M_i$$

.

The total number of Fourier series terms in $H_a^k(\omega)$ is $L_0/\prod_{i=1}^{K} M_i$

Therefore, the total number of Fourier series terms for a K-stage recursion $L(k)$ is

$$L(k) = \frac{L_0}{\prod_{i=1}^{K}} + 4\beta L_0 K\sum_{i=1} M_i \tag{5.5}$$

Differentiating L(k) with respect to M_i, $i = 1, \cdots, K$, and equating the derivatives to zero, we have

$$M_1 = M_2 = \cdots = M_k = M_{opt}(K) \tag{5.6}$$

where

$$M_{opt}(K) = (4\beta)^{-1/(K+1)} \tag{5.7}$$

The corresponding value of $L(k)$ denoted by $L_{min}(k)$ is given by

$$\begin{aligned} L_{min}(k) &= (K+1)4^{K/(K+1)}\beta^{K/(K+1)}L_0 \\ &= (K+1)4^{K/(K+1)}\Theta(\delta_1, \delta_2)\beta^{K/(K+1)} \end{aligned} \tag{5.8}$$

Where $\Theta(\delta_1, \delta_2)$ is a function of δ_1 and δ_2 and is independent of β. Thus, the complexity is proportional to the inverse of the $(K+1)^{th}$ root of β . From (5.8), it can be shown that K satisfies

$$\beta_b(K) < \beta < \beta_b(K-1) \tag{5.9}$$

where

$$\beta_b(K) = \frac{1}{4}left(\frac{K+1}{K+2}right)^{(K+1)(K+2)} \tag{5.10}$$

This leads to the bound

$$\left(\frac{K+1}{K}\right)^K \leq M_{opt}(K) \leq \left(\frac{K+1}{K+2}\right)^{K+2} \tag{5.11}$$

We shall examine $\left(\frac{K+1}{K}\right)^K$ and $\left(\frac{K+1}{K+2}\right)^{K+2}$ separately. When $K = 1$, $\left(\frac{K+1}{K}\right)^K = 2$ and $\left(\frac{K+1}{K+2}\right)^{K+2} = 3.375$. The value of $\left(\frac{K+1}{K}\right)^K$ increases as K increases. When K is very large,

$$\begin{aligned} \left(\frac{K+1}{K}\right)^K &= 1 + K\left(\frac{1}{K}\right) + \frac{K(K-1)}{2!}\left(\frac{1}{K}\right)^2 + \frac{K(K-1)(K-2)}{3!}\left(\frac{1}{K}\right)^3 + \cdots \\ &= 1 + 1 + \frac{1}{2!}\left(\frac{K-1}{K}\right) + \frac{1}{3!}\frac{(K-1)}{K}\frac{(K-2)}{K} + \cdots \\ &= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots \qquad when \;\; K \to \infty \\ &= e \end{aligned} \tag{5.12}$$

[Note:$(1+x)^K = 1 + Kx + \frac{K(K-1)}{2!}x^2 + \frac{K(K-1)(K-2)}{3!}x^3 + \cdots$ and $e^x = 1 + x + \frac{1)}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$]

The value of $\left(\frac{K+2}{K+1}\right)^{K+2}$ decreases as K increases. When K is very large,

$$\left(\frac{K+2}{K+1}\right)^{K+2} = 1+(R+1)\left(\frac{1}{R}\right)+\frac{(R+1)R)}{2!}\left(\frac{1}{R}\right)^2+\frac{(R+1)R(R-2)}{3!}\left(\frac{1}{R}\right)^3+\cdots$$

$$= 1 + \frac{R+1}{R} + \frac{1}{2!}\frac{R+1}{R}\frac{R}{R} + \frac{1}{3!}\frac{R+1}{R}\frac{R}{R}\frac{R-1}{R} + \cdots \qquad (5.13)$$

where $R = K + 1$. Hence,

$$\left(\frac{K+2}{K+1}\right)^{K+2} = 1 + x + \frac{1)}{2!}x^2 + \frac{1}{3!}x^3 + \cdots \qquad when \;\; K \to \infty$$

$$= e \qquad (5.14)$$

From (5.11), (5.12), and (5.14), as $K \to \infty$,$e \leq M_{opt}(K) \leq e$. Thus, for the design of very sharp filter, choose $M_{opt}(K) \approx 3$ since $M_{opt}(K)$ must be an integer. Some of the published materials on FRM approach may be found in the references.

6 CONCLUSION

The FRM approach for minimizing the number of nonzero Fourier series terms for function approximation is presented.

References

L. R. Rabiner, B. Gold (1975), *Theory and Application of Digital Signal Processing*, Prentice-Hall Inc., Chapter 5, pp. 295 - 355.

Y. C. Lim (1986a), Frequency-Response Masking Approach for the Synthesis of Sharp Linear Phase Digital Filter, *IEEE Transactions on Circuits and Systems*, Vol. CAS-33, pp. 357-364.

G. Rajan, Y. Neuvo, S. K. Mitra (1988), On the design of sharp cutoff wideband fir filters with reduced arithmetic complexity, *IEEE Transactions on Circuits and Systems*, Vol. 35, pp. 1447-1454.

Y. C. Lim, Y. Lian (1993), The Optimum Design of One- and Two-Dimensional FIR Filters Using the Frequency Response Masking Technique, *IEEE Transactions on Circuits and Systems II*, Vol. CAS-40, pp. 88-95.

J. H. Lee, C. K. Chen (1993), Design of sharp FIR filters with prescribed group delay, in *Proc. IEEE International Symposium on Circuits and Systems*, Vol. 1, pp. 92-95.

Y. C. Lim, Y. Lian (1994), Frequency-response masking approach for digital filter design: complexity reduction via masking filter factorization, *IEEE Transactions on Circuits and Systems II*, Vol. CAS-II-41, No. 8, pp. 518-525.

M. G. Bellanger (1996), Improved Design of Long FIR Filters Using the Frequency Masking Technique, in *Proc. IEEE Int. Conf. Acoust. Speech. Signal Processing*, pp. 1272-1275.

L. C. R. Barcellos, S. L. Netto, P. S. R. Diniz (2003), Optimization of FRM filters using the WLS-Chebyshev approach, *Circuits, Systems, and Signal Processing*, Vol. 22, No.2, pp. 99-113.

T. Saramäki, Y. C. Lim (2003), Use of the Remez algorithm for designing FRM based FIR filters, *Circuits, Systems, and Signal Processing*, Vol. 22, No.2, pp.77-97.

W. S. Lu, T. Hinamoto (2003a), Optimal Design of Frequency-Response-Masking Filters Using Semidefinite Programming, *IEEE Transactions on Circuits and Systems I*, Vol. 50, pp. 557-568.

Y. Lian, C. Z. Yang (2003), Complexity reduction by decoupling the masking filters from bandedge shaping filter in FRM technique, *Circuits, Systems, and Signal Processing*, Vol. 22, No. 2, pp. 115-135.

Y. Lian (2003), Complexity reduction for FRM based FIR filters using the prefilter-equalizer technique, *Circuits, Systems, and Signal Processing*, Vol. 22, No. 2, pp. 137-155.

T. Saramäki, J. Yli-Kaakinen, H. Johansson (2003), Optimization of Frequency-Response-Masking Based FIR Filters, *Circuits, Systems, and Computers*, Vol. 12, No. 5, pp. 563-590.

W. R. Lee, V. Rehbock, K. L. Teo (2003), Frequency-Response Masking Based FIR Filter Design with Power-of-Two Coefficients and Suboptimum PWR, *Circuits, Systems, and Computers*, Vol. 12, No. 5, pp. 591-600.

O. Gustafsson, H. Johansson, L. Wanhammar (2003a), Single Filter Frequency-Response Masking FIR Filter, *Circuits, Systems, and Computers*, Vol. 12, No. 5, pp. 601-630.

W. S. Lu, T. Hinamoto (2003b), Optimal design of IIR Frequency-Response-Masking filters Using Second-Order Cone Programming, *IEEE Transactions on Circuits and Systems I*, Vol. 50, No. 11, pp. 1401-112.

W. R. Lee, V. Rehbock, K. L. Teo, L. Caccetta (2004), A weighted least-square-based approach to FIR filter design using the frequency-response masking technique, *IEEE Signal Processing Letters*, Vol. 11, pp. 593-596.

A. Usman, S.A. Khan (2004), Simulation of frequency response masking approach for FIR filter design, *WSEAS Transactions on Systems*, Vol. 3, No. 10, pp. 2919-24.

Y. J. Yu, K. L. Teo, Y. C. Lim, G. H. Zhao (2005), Extrapolated Impulse Response Filter and its Application in the Synthesis of Digital Filters Using

the Frequency-Response Masking Technique, *Signal Processing (Elsevier)*, Vol. 85/3, pp. 581-590.

J. X. Rodrigues, K.R. Pai (2005), Modified linear phase frequency response masking FIR filter, in *Proc. 4th International Symposium on Image and Signal Processing and Analysis*, pp. 434-439.

L. Cen, Y. Lian (2006), Hybrid Genetic Algorithm for the Design of Modified Frequency-Response Masking Filters in a Discrete Space, *Circuits, Systems, and Signal Processing*, Vol. 25, No.2, pp. 153-174.

T. Saramäki, Y. C. Lim, R. Yang (1995), The Synthesis of Half-Band Filter Using Frequency-Response Masking Technique, *IEEE Transactions on Circuits and Systems II*, Vol. 42, pp. 58-60.

Y. Lian (1995), The optimum design of half-band filter using multi-stage frequency -response masking technique, *Signal Processing (Elsevier Science)*, Vol. 44, No. 3, pp. 369-372.

C. T. Thia, Y. Lian (2004), The implementation of a high-speed 645-tap equivalent half-band FIR filter using the frequency response masking technique, in *Proc. IEEE TENCON 2004*, Vol. 1, pp. 13-16.

Y. C. Lim, S. H. Low (1998), Frequency-response masking approach for the synthesis of sharp Two-dimensional diamond-shaped filters, *IEEE Transactions on Circuits and Systems II*, Vol. CAS-II-45, No. 12, pp. 1573-1584.

H. Johansson, L. Wanhammar (2000), High-speed recursive digital filters based on the frequency-response masking approach, *IEEE Transactions on Circuits and Systems II*, Vol. 47, pp. 48-61.

M. D. Lutovac, L. D. Milic (2001), IIR filters based on frequency-response masking approach, in *Proc. 5th International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Service*, Vol. 1, Pt. 1, pp. 163-170.

O. Gustafsson, H. Johansson, L. Wanhammar (2003b), Single filter frequency masking high-speed recursive digital filters, *Circuits, Systems, and Signal Processing*, Vol. 22, No. 2, pp. 219-238.

H. H. Chen, S. C. Chan, K. L. Ho (2004), A semi-definite programming (SDP) method for designing IIR sharp cut-off digital filters using frequency-response masking, in *Proc. IEEE International Symposium on Circuits and Systems*, Vol. 3, pp. III 149-152.

J. W. Lee, Y. C. Lim (2002), Efficient implementation of real filter banks using frequency response masking techniques, in *Proceedings of the 2002 IEEE Asia Pacific Conference on Circuits and Systems*, Vol. 1, pp. 69-72.

L. Rosenbaum, P. Lowenborg, H. Johansson (2003), Cosine and sine modulated FIR filter banks utilizing the frequency-response masking approach, in *Proc. IEEE International Symposium on Circuits and Systems*, Vol. 3, pp. III 882-885.

H. Johansson, T. Saramäki (2003), Two-channel FIR filter banks utilizing the FRM approach, *Circuits, Systems, and Signal Processing*, Vol. 22, No. 2, pp. 157-192.

M. B. Furtado, P. S. R. Diniz, S. L. Netto (2003), Optimized prototype filter based on the FRM approach for cosine-modulated filter banks, *Circuits, Systems, and Signal Processing*, Vol. 22, No. 2, pp. 193-210.

S. L. Netto, L. C. R. Barcellos, P. S. R. Diniz (2003), Efficient Design of Narrowband Cosine-Modulated Filter Banks Using a Two-Stage Frequency-Response Masking Approach, *Circuits, Systems, and Computers*, Vol. 12, No. 5, pp. 631-642.

R. Bregovic, T. Saramäki (2005), Design of two-channels FIR filterbanks with rational sampling factors using the FRM technique, in *Proc. IEEE International Symposium on Circuits and Systems*, Vol. 2, pp. 1098-1101.

Y. C. Lim, R. Yang (2005), On the Synthesis of Very Sharp Decimators and Interpolators Using the Frequency-Response Masking Technique, *IEEE Transactions on Signal Processing*, Vol. SP-53, No. 4, pp. 1387-1397.

Hakan Johansson (2006), Two Classes of Frequency-Response Masking Linear-Phase FIR Filters for Interpolation and Decimation, *Circuits, Systems, and Signal Processing*, Vol. 25, No. 2, pp. 175-200.

Y. C. Lim, Y. J. Yu (2005), Synthesis of Very Sharp Hilbert Transformer Using the Frequency-Response Masking Technique, *IEEE Transactions on Signal Processing*, Vol. SP-53, pp. 2595-2597.

Y. C. Lim, Y.J. Yu, T. Saramäki (2005), Optimum Masking Levels and Coefficient Sparseness for Hilbert Transformers and Half-Band Filters Designed Using the Frequency-Response Masking Technique, *IEEE Transactions on Circuits and Systems I*, Vol. CAS-I-52, No. 11, pp. 2444-2453.

Y. C. Lim, Y. J. Yu, H. Q. Zheng, S. W. Foo (2003), FPGA Implementation of digital filters synthesized using the FRM technique, *Circuits, Systems, and Signal Processing*, Vol. 22, No. 2, pp. 211-218.

Y. Lian (2003), A Modified Frequency Response Masking Structure for High-Speed FPGA Implementation of Programmable Sharp FIR Filters, *Circuits, Systems, and Computers*, Vol. 12, No. 5, pp. 643-654.

W. Boonkumklao, Y. Miyanaga, K. Dejhan (2003), A flexible architecture for digital signal processing, *IEICE Transactions on Information and Systems*, Pt. E86D, pp. 2179-2186.

P. S. R. Diniz, L. C. R. de Barcellos, S. L. Netto (2004), Design of High-Resolution Cosine-Modulated Transmultiplexers with Sharp Transition Band, *IEEE Transactions on Signal Processing*, Vol. 52, pp. 1278-1288.

Y. Lian, J.H. Yu (2004), The reduction of noises in ECG signal using a frequency response masking based FIR filter, in *Proc. 2004 IEEE International Workshop on Biomedical Circuits and Systems*, pp. S2/4 17-20.

Y. Lian, Y. Wei (2005), A computationally efficient nonuniform FIR digital filter bank for hearing aids, *IEEE Transactions on Circuits and Systems I*, Vol. 52, No. 12, pp. 2754-2762.

Y. C. Lim (1986b), A digital filter bank for digital audio systems, *IEEE Transactions on Circuits and Systems*, Vol. CAS-33, No. 8, pp. 848-849.

R. H. Yang, S. B. Chiah, W. Y. Chan (1996), Design and implementation of a digital audio tone control unit using an efficient FIR filter structure, in *Proc. IEEE Region 10 Annual International Conference*, Vol. 1, pp. 273-277.

C.S. Lin, C. Kyriakakis (2003), Frequency response masking approach for designing filter banks with rational sampling factors, in *Proc. 2003 IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, pp. 99-102.

S. W. Foo, W. T. Lee (2003), Application of Fast Filter Bank for Transcription of Polyphonic Signals, *Circuits, Systems, and Computers*, Vol. 12, No. 5, pp. 654-674.

S. W. Foo, W. T. Lee, Recognition of piano notes with the aid of FRM filters, in *Proc. International Symposium on Control, Communications and Signal Processing*, pp. 409-413.

N. Hayasaka, N. Wada, S. Yoshizawa, Y. Miyanaga (2004), A robust speech recognition system using FRM running spectrum filtering, in *Proc. International Symposium on Control, Communications and Signal Processing*, pp. 401-404.

Y. Liu, Z. Lin (2006), On the Applications of the Frequency-Response Masking Technique in Array Beamforming, *Circuits, Systems, and Signal Processing*, Vol. 25, No. 2, pp. 201-224.

Y. Zhang, W. Wu, B. Tian (2003), A novel sharp-cutoff FIR filter design technique and its application in software radio, in *Proc. International Conference on Communication Technology*, Vol. 2, pp. 1821 - 1825.

H. Saleh, E. Zimmermann, G. Brandenburg, H. Halling (2001), Efficient FPGA-based multistage two-path decimation filter for noise thermometer, in *Proc. The 13th International Conference on Microelectronics*, pp. 161-164.

Y. C. Lim, Y. J. Yu, K. L. Teo, T. Saramäki (2007), FRM Based FIR Filters with Optimum Finite Word Length Performance, *IEEE Transactions on Signal Processing*, Vol. SP-55, No. 6, pp. 2914-2924.

R. Bregović, Y. C. Lim, T. Saramäki (2008), Frequency-Respons Masking-Based Design of Nearly Perfect-Reconstruction Two-Channel FIR Filterbanks With Rational Sampling Factors, *IEEE Transactions on Circuits and Systems-I*, Vol. CAS-I-55, No. 7, pp. 2002-2012.

K. M. Tsui, S. C. Chan, Y. C. Lim (2008), Design of Multi-Plet Perfect Reconstruction Filter Banks Using Frequency-Response Masking Technique, *IEEE Transactions on Circuits and Systems-I*, Vol. CAS-I-55, No. 9, pp. 2707-2715.

Y. J. Yu, Y. C. Lim, D. Shi (2009), Low Complexity Design of Variable Band-edge Linear Phase FIR Filters with Sharp Transition Band, *IEEE Transactions on Signal Processing*, Vol. SP-57, No. 4, pp. 1328-1338.

Y. C. Lim, J. W. Lee, S. W. Foo (2010), Quality Analog Scramblers using Frequency Response Masking Filter Banks, *Circuits, Systems and Signal Processing*. Vol. 29, No. 1, pp. 135-154.