AN ITERATIVE METHOD FOR OPTIMIZING FIR FILTERS SYNTHESIZED USING THE TWO-STAGE FREQUENCY-RESPONSE MASKING TECHNIQUE

Ya Jun Yu [1*] *Tapio Saramäki* [2] and *Yong Ching Lim* [1]

[1] Dept. of Electrical & Computer Engineering
National University of Singapore
Singapore

[2] Institute of Signal Processing
Tampere University of Technology
Tampere, Finland

ABSTRACT

An efficient technique for drastically reducing the number of multipliers and adders in narrow-transition band linear-phase finite-impulse response filters is to use one-stage or multistage frequency-response masking (FRM) approaches as originally introduced by Lim. It has been observed recently that for the one-stage FRM approach the filter complexity can be considerably reduced by iteratively optimizing the periodic filter and masking filters by properly sharing the frequency-response-shaping responsibilities in their respective frequency regions. In this paper, a similar iterative method is applied to the two-stage FRM structure to reduce the filter complexity even more. An example taken from the literature is included illustrating that the number of adders and multipliers for the resulting filters are less than 75 percent compared with the original designs.

1. INTRODUCTION

The frequency-response masking (FRM) approach [1–5] is one of the most efficient techniques for synthesizing narrow-transition band linear-phase finite-impulse response (FIR) digital filters. It produces filters with very sparse coefficients and, hence, results in tremendous savings in the number of multipliers and adders when compared to the conventional direct-form realization. The price to be paid for the enormous reduction in the computational complexity is a slight increase in the filter order. Recently, an iterative design scheme [4] has been developed for effectively optimizing the sub-filters. In this method, the periodic filter and masking filters share properly the frequency-response-shaping responsibilities by taking care of their respective frequency regions. The filters obtained by this scheme are good sub-optimum solutions to the problem and start-up solutions for further simultaneously optimization. It has been shown that the number of adders and multipliers of the resulting filters are less than 80 percent compared to those filters obtained by using the original design schemes [1,2].

In order to reduce the filter complexity even further, a similar iterative design scheme is applied in this paper to the two-stage FRM technique, where the periodic filter in the one-stage FRM structure is implemented by using another FRM structure. It is shown, by means of an example, that the number of adders and multipliers of the resulting filters are less than 75 percent compared with those two-stage filters obtained by using the original design schemes [1, 2].

2. ONE-STAGE FRM APPROACH

In the one-stage FRM approach, the overall linear-phase FIR filter transfer function is constructed as follows [1–3]:

$$H(z) = F(z^L)G_1(z) + [z^{-LN_F/2} - F(z^L)]G_2(z), \quad (1)$$

where

$$F(z^L) = \sum_{n=0}^{N_F} f(n)z^{-nL} \quad (2)$$

and

$$G_k(z) = z^{-M_k} \sum_{n=0}^{N_k} g_k(n)z^{-n} \quad \text{for} \quad k = 1, 2. \quad (3)$$

Here, the impulse response coefficients $f(n)$, $g_1(n)$, and $g_2(n)$ possess an even symmetry. N_F is even, whereas both N_1 and N_2 are either even or odd. For $N_1 \geq N_2$, $M_1 = 0$ and $M_2 = (N_1 - N_2)/2$, whereas for $N_1 \leq N_2$, $M_1 = (N_1 - N_2)/2$ and $M_2 = 0$. These selections guarantee that the delays for both of the terms of $H(z)$ are equal.

The zero-phase frequency response of $H(z)$ is expressible as

$$H(\omega) = F(L\omega)G_1(\omega) + [1 - F(L\omega)]G_2(\omega). \quad (4)$$

If $F(z)$ and $z^{-N_F/2} - F(z)$ form a lowpass-highpass filter pair with edges at θ and ϕ, then $F(z^L)$ and $z^{-LN_F/2} - F(z^L)$ provide various transition bands of width of $(\phi - \theta)/L$ that are only $(1/L)$th of that of $F(z)$ and $z^{-N_F/2} - F(z)$. The role of the masking filters $G_1(z)$ and $G_2(z)$ is to attenuate the unnecessary frequency components of $F(z^L)$ and $z^{-LN_F/2} - F(z^L)$ in order to achieve the desired overall frequency response [1, 2].

For a lowpass overall transfer function $H(z)$ with the passband and stopband edges at ω_p and ω_s, and the given L, one of following two cases (but not both) may yield a set of θ and ϕ satisfying $0 \leq \theta < \phi \leq \pi$. In the first case, referred to as Case A, the parameters for the one-stage design are given by [1–3]

$$l = \lfloor L\omega_p/(2\pi) \rfloor, \; \theta = L\omega_p - 2l\pi, \; \phi = L\omega_s - 2l\pi \quad (5)$$

and in the second case, denoted by Case B, by

$$l = \lceil L\omega_s/(2\pi) \rceil, \; \theta = 2l\pi - L\omega_s, \; \phi = 2l\pi - L\omega_p. \quad (6)$$

Here, $\lfloor x \rfloor$ is the largest integer less than or equal to x, and $\lceil x \rceil$ is the smallest integer larger than or equal to x.

In [4], an iterative scheme for optimizing the sub-filters has been proposed. The key idea is to design $F(z^L)$ to provide the desired overall filter performance on $[\Omega_{p1}, \; \Omega_{p2}] \cup [\Omega_{s1}, \; \Omega_{s2}]$, whereas $G_1(z)$ and $G_2(z)$ provide the overall filter performance on $[0, \; \Omega_{p1}] \cup [\Omega_{s2}, \; \pi]$, where

$$\begin{array}{llll} & \text{Case A Design} & \text{Case B Design} & \\ \Omega_{p1} & = 2l\pi/L & = (2l-1)\pi/L & \\ \Omega_{p2} & = (2l\pi+\theta)/L & = (2l\pi-\phi)/L & = \omega_p \\ \Omega_{s1} & = (2l\pi+\phi)/L & = (2l\pi-\theta)/L & = \omega_s \\ \Omega_{s2} & = (2l+1)\pi/L & = 2l\pi/L. & \end{array} \quad (7)$$

The iterative algorithm is used by alternately designing the periodic filter $F(z^L)$ and the masking filters $G_1(z)$ and $G_2(z)$ to take care of the frequency-response-shaping responsibilities in

*This research was performed while the author was visiting the Institute of Signal Processing, Tampere University of Technology, Finland.

0-7803-7762-1/03/$17.00 ©2003 IEEE

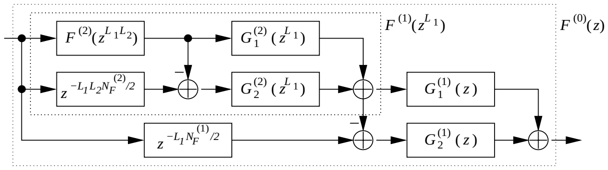

Fig. 1. The structure of the filter synthesized using the two-stage FRM technique.

their respective frequency regions until the difference between successive overall solutions is within the given tolerance limits. It has been shown that, for the filters designed using this technique, the masking filter orders reduce to be approximate 60 percent of those of the original designs.

3. TWO-STAGE FRM APPROACH

If the order of $F(z)$ in (1) is too high, its complexity can be reduced by implemented it by using another FRM structure. This results in the two-stage FRM approach [1–3], as shown in Fig. 1. In this case, the overall transfer function $H(z)$ is expressible as

$$H(z) \equiv F^{(0)}(z) = F^{(1)}(z^{L_1})G_1^{(1)}(z) + [z^{-L_1 N_F^{(1)}/2} - F^{(1)}(z^{L_1})]G_2^{(1)}(z), \quad (8)$$

where

$$F^{(1)}(z) = F^{(2)}(z^{L_2})G_1^{(2)}(z) + [z^{-L_2 N_F^{(2)}/2} - F^{(2)}(z^{L_2})]G_2^{(2)}(z).$$

Here, $F^{(2)}(z)$, $G_1^{(2)}(z)$, $G_2^{(2)}(z)$, $G_1^{(1)}(z)$, and $G_2^{(1)}(z)$ are the filters to be designed, and $N_F^{(2)}$, $N_1^{(2)}$, $N_2^{(2)}$, $N_1^{(1)}$, and $N_2^{(1)}$ are their orders, respectively. In order to obtain a desired overall solution, $N_F^{(2)}$, $N_1^{(2)}$, and $N_2^{(2)}$ have to be even.

When designing a lowpass overall transfer function $H(z)$ with the passband and stopband edges at ω_p and ω_s for the given L_m's for $m = 1, 2$, only one or none of the following four cases: Case A$^{(2)}$A$^{(1)}$, Case B$^{(2)}$A$^{(1)}$, Case A$^{(2)}$B$^{(1)}$, and Case B$^{(2)}$B$^{(1)}$, results in $F^{(m)}(z)$'s for $m = 1, 2$ so that their passband and stopband edges, denoted by θ_m and ϕ_m, respectively, satisfy $0 \le \theta_m < \phi_m \le \pi$ for $m = 1, 2$. Here,

	Case A$^{(m)}$ Design	Case B$^{(m)}$ Design
l_m	$= \lfloor L_m \omega_{p,m}/(2\pi) \rfloor$	$= \lceil L_m \omega_{s,m}/(2\pi) \rceil$
θ_m	$= L_m \omega_{p,m} - 2 l_m \pi$	$= 2 l_m \pi - L_m \omega_{s,m}$
ϕ_m	$= L_m \omega_{s,m} - 2 l_m \pi$	$= 2 l_m \pi - L_m \omega_{p,m}.$

(9)

Here, $\omega_{p,1} = \omega_p$, $\omega_{s,1} = \omega_s$, $\omega_{p,2} = \theta_1$, and $\omega_{s,2} = \phi_1$. Therefore, ω_p and ω_s can be expressed in terms of θ_2, ϕ_2, l_1, l_2, L_1, and L_2 for each case as given by the sixth and seventh rows in Table 1 (see also Fig. 2).

Assuming that the filter orders and the remaining design parameters have been predetermined, the proposed iterative procedure for designing a two-stage FRM filter transfer function $H(z)$ with the passband and stopband ripples of δ_p and δ_s can be carried out as follows [1]:

Step 1: Set $r = 1, \epsilon^r_{G^{(1)}} = \epsilon^r_{G^{(2)}} = \epsilon^r_{F^{(2)}} = 0$. Determine the coefficients of $F^{(2),r}(z)$ to minimize

$$\max_{\omega \in [0,\, \theta_2] \cup [\phi_2,\, \pi]} \left| W(\omega)[F^{(2),r}(\omega) - D(\omega)] \right| . \quad (10)$$

[1]Due to the lack of space, a detailed description of the performance of this procedure will be given in a full-length paper to be published later on.

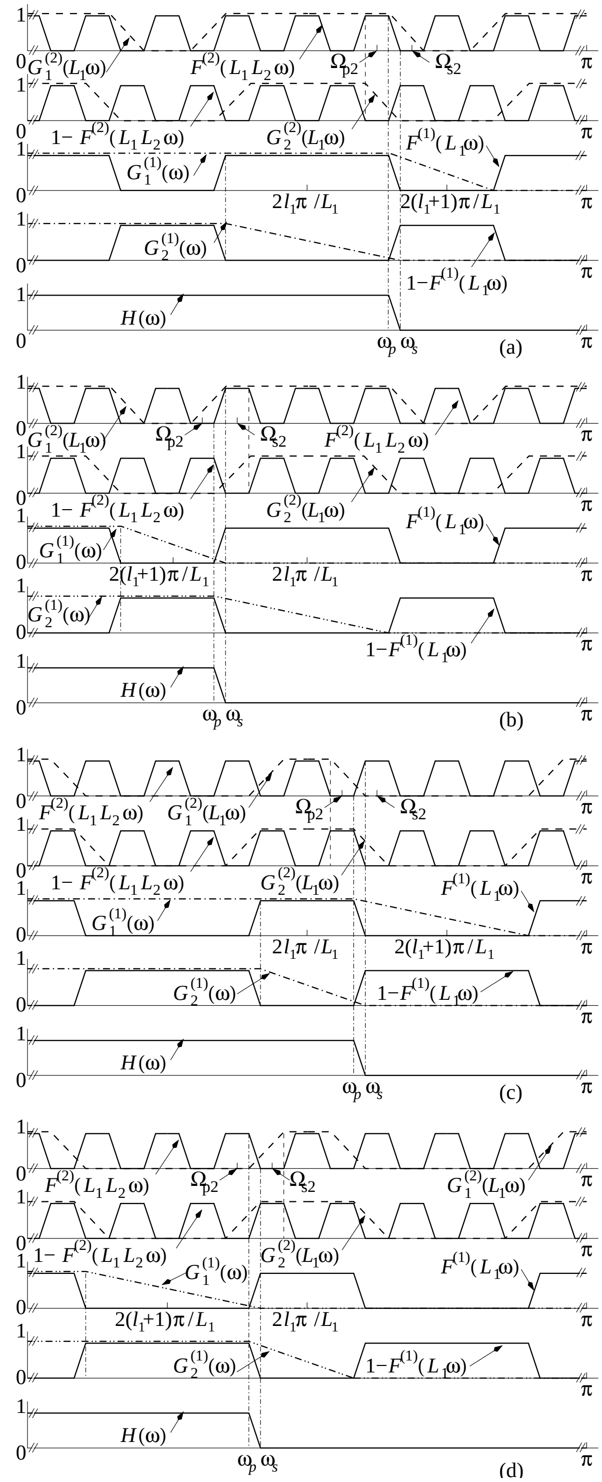

Fig. 2. Four cases for designing a lowpass filter using the two-stage FRM technique. (a) Case A$^{(2)}$A$^{(1)}$. (b) Case A$^{(2)}$B$^{(1)}$. (c) Case B$^{(2)}$A$^{(1)}$. (d) Case B$^{(2)}$B$^{(1)}$. The values of Ω_{p2}, ω_p, ω_s, and Ω_{s2} are given in Table 1 for each case.

Here, the first band is the passpand, where $W(\omega)$ and $D(\omega)$ are equal to unity. The second band is the stopband, where $W(\omega)$ is

Table 1. The passband and stopband edges of the two masking filters and the main periodic filter used in the proposed iterative algorithm for synthesizing efficient two-stage FRM based linear-phase FIR filters

	Case A$^{(2)}$A$^{(1)}$	A$^{(2)}$Case B$^{(1)}$	Case B$^{(2)}$A$^{(1)}$	Case B$^{(2)}$B$^{(1)}$
Ω_p	$2l_2\pi/L_2$		$(2l_2-1)\pi/L_2$	
Ω_s	$(2l_2+1)\pi/L_2$		$2l_2\pi/L_2$	
Ω_{p1}	$2l_l\pi/L_1$	$(2l_1-1)\pi/L_1$	$2l_l\pi/L_1$	$(2l_1-1)\pi/L_1$
Ω_{p2}	$(2l_1\pi+2l_2\pi/L_2)/L_1$	$[2l_1\pi-(2l_2+1)\pi/L_2]/L_1$	$[2l_1\pi+(2l_2-1)\pi/L_2]/L_1$	$(2l_1\pi-2l_2\pi/L_2)/L_1$
$\Omega_{p3}=\omega_p$	$[2l_1\pi+(2l_2\pi+\theta_2)/L_2]/L_1$	$[2l_1\pi-(2l_2\pi+\phi_2)/L_2]/L_1$	$[2l_1\pi+(2l_2\pi-\phi_2)/L_2]/L_1$	$[2l_1\pi-(2l_2\pi-\theta_2)/L_2]/L_1$
$\Omega_{s1}=\omega_s$	$[2l_1\pi+(2l_2\pi+\phi_2)/L_2]/L_1$	$[2l_1\pi-(2l_2\pi+\theta_2)/L_2]/L_1$	$[2l_1\pi+(2l_2\pi-\theta_2)/L_2]/L_1$	$[2l_1\pi-(2l_2\pi-\phi_2)/L_2]/L_1$
Ω_{s2}	$[2l_1\pi+(2l_2+1)\pi/L_2]/L_1$	$(2l_1\pi-2l_2\pi/L_2)/L_1$	$(2l_1\pi+2l_2\pi/L_2)/L_1$	$[2l_1\pi-(2l_2-1)\pi/L_2]/L_1$
Ω_{s3}	$(2l_1+1)\pi/L_1$	$2l_1\pi/L_1$	$(2l_1+1)\pi/L_1$	$2l_1\pi/L_1$

equal to δ_p/δ_s and $D(\omega)$ is equal to zero. In the sequel, the same desired and weighting functions are used.

Step 2: Determine the coefficients of $G_k^{(2),r}(z)$ for $k = 1,2$ to minimize

$$\max_{\omega\in[0,\,\Omega_p]\cup[\Omega_s,\,\pi]}\left|W(\omega)[H^r_{G^{(2)}}(\omega)-D(\omega)]\right|, \tag{11}$$

where

$$H^r_{G^{(2)}}(\omega)=F^{(2),r}(L_2\omega)G_1^{(2),r}(\omega)+[1-F^{(2),r}(L_2\omega)]G_2^{(2),r}(\omega)$$

and the values of Ω_p and Ω_s are given in Table 1.

Step 3: Set $r = r+1$. Determine the coefficients of $G_k^{(1),r}(z)$ for $k = 1,2$ to minimize

$$\epsilon^r_{G^{(1)}}=\max_{\omega\in[0,\,\Omega_{p1}]\cup[\Omega_{s3},\,\pi]}\left|W(\omega)[H^r_{G^{(1)}}(\omega)-D(\omega)]\right|, \tag{12}$$

where

$$\begin{aligned}H^r_{G^{(1)}}(\omega)&=\Big\{F^{(2),r-1}(L_1L_2\omega)G_1^{(2),r-1}(L_1\omega)\\&+\left[1-F^{(2),r-1}(L_1L_2\omega)\right]G_2^{(2),r-1}(L_1\omega)\Big\}G_1^{(1),r}(\omega)\\&+\Big\{1-F^{(2),r-1}(L_1L_2\omega)G_1^{(2),r-1}(L_1\omega)\\&-\left[1-F^{(2),r-1}(L_1L_2\omega)\right]G_2^{(2),r-1}(L_1\omega)\Big\}G_2^{(1),r}(\omega)\end{aligned} \tag{13}$$

and the values of Ω_{p1} and Ω_{s3} are given in Table 1.

Step 4: Set $F^{(2),r}(\omega) = F^{(2),r-1}(\omega)$, $\epsilon^r_{F^{(2)}} = \epsilon^{r-1}_{F^{(2)}}$, $G_k^{(2),r}(\omega) = G_k^{(2),r-1}(\omega)$ for $k = 1,2$, and $\epsilon^r_{G^{(2)}} = \epsilon^{r-1}_{G^{(2)}}$.

Step 5: Set $r = r+1$, $G_k^{(1),r}(\omega) = G_k^{(1),r-1}(\omega)$ for $k = 1,2$, and $\epsilon^r_{G^{(1)}} = \epsilon^{r-1}_{G^{(1)}}$. Determine the coefficients of $G_k^{(2),r}(z)$ for $k = 1,2$ to minimize

$$\epsilon^r_{G^{(2)}}=\max_{\omega\in[\Omega_{p1},\,\Omega_{p2}]\cup[\Omega_{s2},\,\Omega_{s3}]}\left|W(\omega)[H^r_{G^{(2)}}(\omega)-D(\omega)]\right|, \tag{14}$$

where

$$\begin{aligned}H^r_{G^{(2)}}(\omega)&=F^{(2),r-1}(L_1L_2\omega)\Big[G_1^{(1),r}(\omega)\\&-G_2^{(1),r}(\omega)\Big]G_1^{(2),r}(L_1\omega)+\left[1-F^{(2),r-1}(L_1L_2\omega)\right]\\&\times\Big[G_1^{(1),r}(\omega)-G_2^{(1),r}(\omega)\Big]G_2^{(2),r}(L_1\omega)+G_2^{(1),r}(\omega)\end{aligned} \tag{15}$$

and the values of Ω_{p2} and Ω_{s2} are given in Table 1.

Step 6: Determine the coefficients of $F^{(2),r}(z)$ to minimize

$$\epsilon^r_{F^{(2)}}=\max_{\omega\in[\Omega_{p2},\,\Omega_{p3}]\cup[\Omega_{s1},\,\Omega_{s2}]}\left|W(\omega)[H^r_{F^{(2)}}(\omega)-D(\omega)]\right|, \tag{16}$$

where

$$\begin{aligned}H^r_{F^{(2)}}(\omega)&=F^{(2),r}(L_1L_2\omega)\Big[G_1^{(2),r}(L_1\omega)\\&-G_2^{(2),r}(L_1\omega)\Big]\Big[G_1^{(1),r}(\omega)-G_2^{(1),r}(\omega)\Big]\\&+G_2^{(2),r}(L_1\omega)\Big[G_1^{(1),r}(\omega)-G_2^{(1),r}(\omega)\Big]+G_2^{(1),r}(\omega)\end{aligned} \tag{17}$$

and the values of Ω_{p3} and Ω_{s1} are given in Table 1.

Step 7: If $|\epsilon^r_{G^{(2)}}-\epsilon^{r-1}_{G^{(2)}}| < \Delta_2$ and $|\epsilon^r_{F^{(2)}}-\epsilon^{r-1}_{F^{(2)}}| < \Delta_2$ or $\epsilon^r_{G^{(2)}} < \epsilon^r_{G^{(1)}}$ and $\epsilon^r_{F^{(2)}} < \epsilon^r_{G^{(1)}}$, where Δ_2 is a prescribed tolerance, then go to Step 8. Otherwise, go to Step 5.

Step 8: If $|\epsilon^r_{G^{(1)}}-\epsilon^{r-1}_{G^{(1)}}| < \Delta_1$, $|\epsilon^r_{G^{(2)}}-\epsilon^{r-1}_{G^{(2)}}| < \Delta_1$, and $|\epsilon^r_{F^{(2)}}-\epsilon^{r-1}_{F^{(2)}}| < \Delta_1$, where Δ_1 is another prescribed tolerance being less than Δ_2, then stop. Otherwise, go to Step 3.

Step 1 can be accomplished by using the Remez algorithm, whereas Steps 2, 3, 5 and 6 can be implemented using linear programming [2].

For the filter design using this technique, good estimates for $N_k^{(m)}$ for $m = 1,2$ and $k = 1,2$ are 60 percent of those for the original two-stage designs, whereas the estimates of $N_F^{(2)}$ are approximately the same for both designs. Thus, L_1 and L_2 can be selected by using an exhaust search based on this observation to achieve the lowest implementation complexity (usually, the lowest number of multipliers).

4. NUMERICAL EXAMPLE

This section illustrates, by means of an example, the efficiency of the filters obtained by applying the proposed technique compared to those obtained using the earlier two-stage design schemes.

Consider the specifications [3–5]: $\omega_p = 0.4\pi$, $\omega_s = 0.402\pi$, $\delta_p = 0.01$, and $\delta_s = 0.001$. For the optimum conventional direct-form FIR filter design, the minimum order to meet the given criteria is 2541, requiring 2541 adders and 1271 multipliers when the coefficient symmetry is exploited.

For original two-stage design, $L_1 = L_2 = 6$ minimizes the number of multipliers required in the implementation [3]. For these values, the overall filter is a Case A$^{(2)}$A$^{(1)}$ design with

[2] It has been observed that the convergence of the above algorithm can be made considerably faster by multiplying the upper passband edges [lower stopband edges] in (12), (14) and (16) by $(1+\alpha)$ $[(1-\alpha)]$. In most cases, $\alpha = 0.01$ is a proper selection.

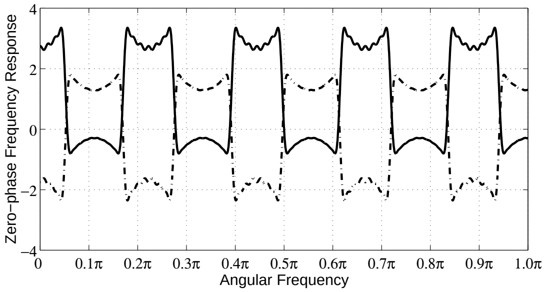

Fig. 3. Responses for $F^{(2)}(L_2\omega)$ (solid line) and $1-F^{(2)}(L_2\omega)$ (dot-dashed line) for the proposed filter for $L_1 = L_2 = 9$.

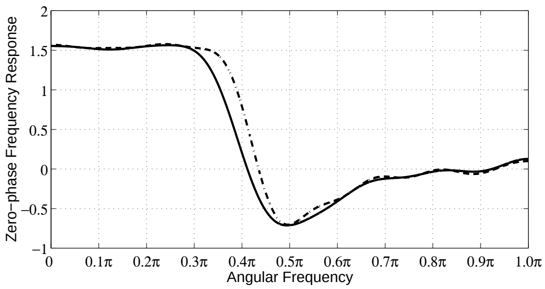

Fig. 4. Responses for $G_1^{(2)}(\omega)$ (solid line) and $G_2^{(2)}(\omega)$ (dot-dashed line) for the proposed filter for $L_1 = L_2 = 9$.

$\theta_1 = 0.4\pi$, $\phi_1 = 0.412\pi$, $\theta_2 = 0.4\pi$, and $\phi_2 = 0.472\pi$. The minimum orders of $F^{(2)}(z)$, $G_1^{(2)}(z)$, $G_2^{(2)}(z)$, $G_1^{(1)}(z)$, and $G_2^{(1)}(z)$ are 74, 28, 36, 26, and 40, respectively. The overall number of multipliers and adders are 107 and 204, respectively. The overall filter order is 2920.

For $L_1 = L_2 = 6$, the best solution resulting when using the proposed iterative scheme is obtained by $N_F^{(2)} = 74$, $N_1^{(2)} = 16$, $N_2^{(2)} = 20$, $N_1^{(1)} = 17$, and $N_2^{(1)} = 23$. For this filter, the number of multipliers and adders are 79 and 150, respectively. The overall filter order reduces to 2807.

For the proposed technique, the overall number of multipliers is minimized by $L_1 = L_2 = 9$. For these values, the overall filter is a Case $\mathrm{B}^{(2)}\mathrm{B}^{(1)}$ design with $\theta_1 = 0.382\pi$, $\phi_1 = 0.4\pi$, $\theta_2 = 0.4\pi$, and $\phi_2 = 0.562\pi$. The best result is obtained by $N_F^{(2)} = 32$, $N_1^{(2)} = 28$, $N_2^{(2)} = 30$, $N_1^{(1)} = 23$, and $N_2^{(1)} = 35$. This filter requires 78 multipliers and 148 adders that are approximately 73 percent of those of the original design for $L_1 = L_2 = 6$.

For the best design with $L_1 = L_2 = 9$, Figs. 3 and 4 show the responses $F^{(2)}(L_2\omega)$ and $1-F^{(2)}(L_2\omega)$; and $G_1^{(2)}(\omega)$ and $G_2^{(2)}(\omega)$, respectively. The responses $F^{(1)}(L_1\omega) = F^{(2)}(L_1L_2\omega)G_1^{(2)}(L_1\omega) + [1-F^{(2)}(L_1L_2\omega)]G_2^{(2)}(L_1\omega)$ and $1-F^{(1)}(L_1\omega)$; and $G_1^{(1)}(\omega)$ and $G_2^{(1)}(\omega)$ are shown in Figs. 5 and 6, respectively, whereas the overall response for $H(\omega) \equiv F^{(0)}(\omega)$ is depicted in Fig. 7.

5. CONCLUSION

An efficient iterative algorithm has been proposed for designing FIR filters using the two-stage frequency-response masking technique. Compared with the original synthesis schemes, the proposed algorithm results in approximately 25 percent savings in the number of multipliers and adders. This is mainly due to approximately 40 percent reductions in the orders of the masking filters, as in the case of iteratively designing one-stage FRM based FIR filters [4].

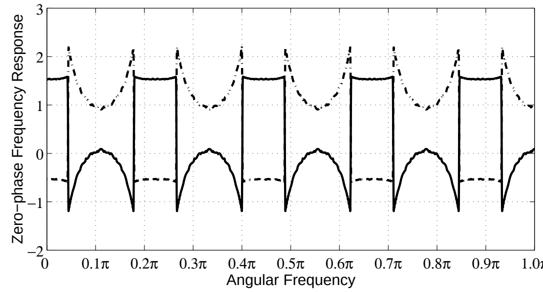

Fig. 5. Responses for $F^{(1)}(L_1\omega)$ (solid line) and $1-F^{(1)}(L_1\omega)$ (dot-dashed line) for the proposed filter for $L_1 = L_2 = 9$.

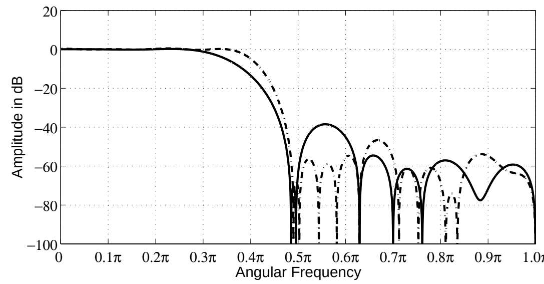

Fig. 6. Responses for $G_1^{(1)}(\omega)$ (solid line) and $G_2^{(1)}(\omega)$ (dot-dashed line) for the proposed filter for $L_1 = L_2 = 9$.

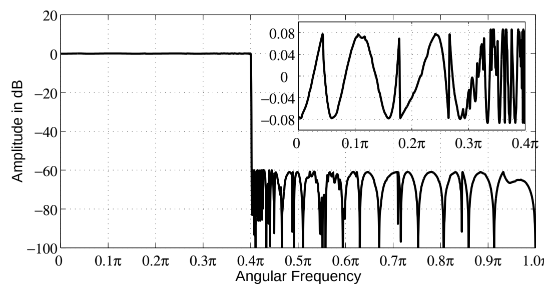

Fig. 7. Response for the proposed overall filter for $L_1 = L_2 = 9$.

6. REFERENCES

[1] Y. C. Lim, "Frequency-response masking approach for the synthesis of sharp linear phase digital filters," *IEEE Trans. Circuits Syst.*, vol. CAS-33, pp. 357-364, Apr. 1986.

[2] Y. C. Lim and Y. Lian, "The optimum design of one- and two-dimensional FIR filters using the frequency-response masking technique," *IEEE Trans. Circuits Syst II.*, vol. 40, pp. 88-95, Feb. 1993.

[3] T. Saramäki and Y. C. Lim, "Use of the Remez algorithm for designing FIR filters utilizing the frequency-response masking approach," in *Proc. IEEE Int. Conf. Circuits Syst.*, Orlando, FL, vol. III, pp. 449-455, 1999.

[4] T. Saramäki and H. Johansson, "Optimization of FIR filters using the frequency-response masking technique," in *Proc. IEEE Int. Conf. Circuits Syst.*, Sydney, Australia, vol. II, pp. 177-180, 2001.

[5] T. Saramäki and J. Yli-Kaakinen, "Optimization of frequency response masking based FIR filters with reduced complexity," in *Proc. IEEE Int. Conf. Circuits Syst.*, Scottdale, Arizona, vol. III, pp. 225-228, 2002.