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Wave digital filters with minimum multiplier for discrete Hilbert transformer realization

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Abstract

This paper describes the design of wave digital filters for discrete Hilbert transformer realization. A method for designing real and complex half-band wave digital filters (WDFs) with a minimum number of multipliers is afforded where the multiplier coefficients for the adaptors are calculated directly from the digital domain transfer functions. This technique uses the equations derived for a class of infinite impulse response (IIR) filter and hence avoids the conventional technique of synthesizing a WDF from an analogue reference filter. The selection of the adaptor configuration is determined from the multiplier coefficient values. A minimum multiplier realization of real half-band WDFs can be obtained to meet the low-pass digital filter magnitude specifications. By applying a transformation equation, a complex half-band WDF suitable for the realization of the discrete Hilbert transformer with a minimum number of multiplier is obtained.

Keywords: Wave digital filters; Hilbert transformer; Minimum multiplier

1. Introduction

Wave digital filters (WDFs) represent a class of digital filters that are closely related to classical filter networks, usually lossless filters inserted between resistive terminations. For every WDF, there corresponds a reference analogue filter from which it is derived. The analogy between a WDF and its reference network is based not on the use of voltages and currents as the signal variables but the traveling wave quantities of classical electric circuits. A variety of WDFs are available and many have found applications in systems requiring digital filters. This is due to the good sensitivity properties of the WDFs obtained from a properly designed analogue filter that is both passive and lossless. This property is preserved in the digital domain by the structures of the WDFs which emulate the analogue filters. The good sensitivity properties translate to reduced accuracy requirement for the coefficient wordlength and a good dynamic range performance [1–3].

Filters are classified according to the characteristics of their frequency response. Real half-band

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filters are usually a pair of filters with one having a low-pass response occupying the lower half of the useful digital frequency band, and the other with a high-pass response occupying the upper half of the useful digital frequency band. These filters are usually used in pairs for applications such as in subband coding. Complex half-band filters, on the other hand, are often used to realize the discrete Hilbert transformer. The discrete Hilbert transformer arises in a variety of practical applications, including inverse filtering, complex representations for real bandpass signals, single side-band modulation techniques and spectral analysis. Both infinite impulse response (IIR) and finite impulse response (FIR) filters can be used to realize these filters [4]. For applications requiring strict linear phase, FIR filters are usually used [5]. However, when the phase requirement is not as stringent, the IIR filter is a better choice as it requires fewer coefficients to realize the same transfer function as an FIR filter [6]. Using IIR filters are especially advantageous in realizing low-cost and low-power devices.

Wave digital filters have been shown to be applicable to the design of real half-band filters [7–11]. Many of the methods rely on deriving the WDF structure from reference analogue lattice filters and the realizations may not be optimum in terms of the number of multipliers required given a filter order. In addition, complex realizations are not shown for these real filters.

For complex half-band WDFs the design technique relies on deriving the WDF structure from reference analogue filters having the complex response [12]. For these filter types, the corresponding WDF structures are obtained using an element to element translation from the analog to the digital domain and interconnecting the digital elements using the appropriate adaptors. A purely digital approach showing the resulting effects in both the magnitude and phase responses has not been shown.

In this paper, a purely digital approach is introduced to design real and complex half-band WDFs. The transfer functions for a class of IIR filter are used to obtain all-pass WDF structures which are then used to obtain real half-band filters. Next, complex half-band filter WDFs are obtained from real half-band WDFs by applying a transformation equation. The adaptor selection is determined from the multiplier coefficient values which may differ for the real and complex WDFs.

This paper is organized in the following manner: Section 2 details the design methodology of real and complex IIR filters, Section 3 presents the design parameters of wave digital filters, Section 4 gives an example of the proposed technique while the conclusion is presented in Section 5.

2. Designing half-band IIR filters

2.1. Real half-band IIR filters

Real half-band filters can be realized with either FIR or IIR filters. For IIR filters, it has been shown [13] that any odd order elliptic lowpass half-band filter, $G(z)$, with a frequency response given by

$$1-\delta_p \leqslant \left|G(e^{j\omega})\right| \leqslant 1 \quad \text{for} \quad 0 \leqslant \omega \leqslant \omega_p, \tag{1a}$$

$$\left|G(e^{j\omega})\right| \leqslant \delta_s, \quad \text{for} \quad \omega_s \leqslant \omega \leqslant \pi, \tag{1b}$$

and satisfying the conditions

$$\omega_p + \omega_s = \pi, \tag{2a}$$

$$(1-2\delta_p)^2 + \delta_s^2 = 1, \tag{2b}$$

can be expressed as

$$G(z) = \tfrac{1}{2}[A_0(z^2) + z^{-1}A_1(z^2)], \tag{3}$$

where $A_0[z]$ and $A_1[z]$ are stable magnitude all-pass filters. δ_p and δ_s are the passband and stopband ripple, respectively, while ω_p and ω_s are the passband and stopband normalized edge frequency, respectively.

For $G(z)$ with real coefficients, the frequency response is

$$G(e^{j\omega}) + G(e^{j(\pi-\omega)}) = 1. \tag{4}$$

The above equality implies that $G(e^{j(\pi/2-\theta)})$ and $G(e^{j(\pi/2+\theta)})$ add up to unity for all θ. Hence, $G(e^{j\omega})$ exhibits symmetry with respect to the half-band frequency, $\pi/2$. This filter has a doubly complementary high-pass transfer function $H_1(z)$ given by

$$H_1(z) = \tfrac{1}{2}[A_0(z^2) - z^{-1}A_1(z^2)]. \tag{5}$$

Fig. 1 shows the realization of the half-band IIR filter based on Eqs. (3) and (5).

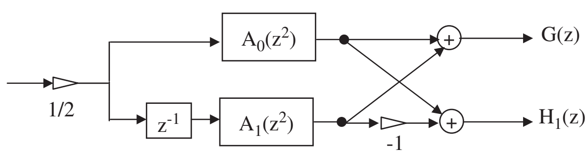

Fig. 1. Realization of a real-half band IIR filter.

2.2. Complex half-band IIR filters

A complex half-band filter can be used to realize the discrete Hilbert transformer for the generation of an analytic signal from a discrete-time real signal. The analytic signal is a complex time function having a Fourier transform that vanishes for negative frequencies. The basic scheme for the generation of an analytic signal $y[n] = y_{re}[n]+y_{im}[n]$ from a real signal $x[n]$ is shown in Fig. 2.

A linear discrete-time system representing an ideal Hilbert transformer has a frequency response corresponding to

$$H_{HT}(\omega) = \begin{cases} -j, & 0<\omega<\pi, \\ j, & -\pi<\omega<0. \end{cases} \tag{6}$$

However, Eq. (6) represents an unrealizable ideal system with an infinite-length two-sided impulse response. An approach to develop a realizable system is by using complex half-band filters. Consider the complex analytic signal

$$y[n] = x[n] + j\hat{x}[n], \tag{7}$$

where $x[n]$ and $\hat{x}[n]$ are real. Its discrete-time Fourier transform (DTFT) $Y(e^{j\omega})$ is given by

$$Y(e^{j\omega}) = X(e^{j\omega}) + j\hat{X}(e^{j\omega}). \tag{8}$$

Since $x[n]$ and $\hat{x}[n]$ are real, their corresponding DTFTs are conjugate symmetric, hence the following can be obtained:

$$X(e^{j\omega}) = \tfrac{1}{2}\left[Y(e^{j\omega}) + Y*(e^{-j\omega})\right], \tag{9}$$

$$j\hat{X}(e^{j\omega}) = \tfrac{1}{2}\left[Y(e^{j\omega}) - Y*(e^{-j\omega})\right]. \tag{10}$$

For an analytic signal, $Y(e^{j\omega})$ is zero for $-\pi \leqslant \omega \leqslant 0$, hence from Eqs. (8)–(10) the following is derived:

$$Y(e^{j\omega}) = \begin{cases} 2X(e^{j\omega}), & 0\leqslant\omega<\pi, \\ 0, & -\pi\leqslant\omega<0. \end{cases} \tag{11}$$

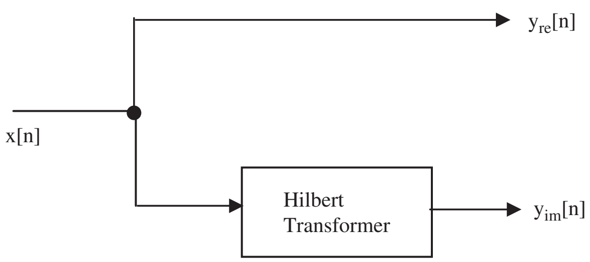

Fig. 2. Generation of an analytic signal using a Hilbert transformer.

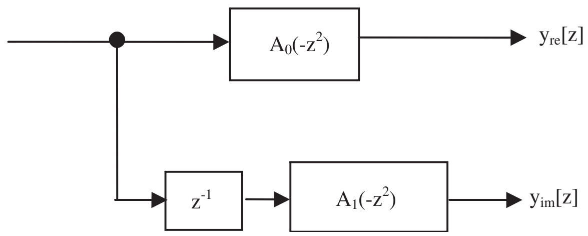

Fig. 3. IIR realization of a complex half-band filter.

The analytic signal can be generated by passing $x[n]$ through a linear discrete-time system with a frequency response $H(e^{j\omega})$ given by

$$H(e^{j\omega}) = \begin{cases} 2, & 0\leqslant\omega<\pi, \\ 0, & -\pi\leqslant\omega<0. \end{cases} \tag{12}$$

2.2.1. Transformation equations

To obtain a complex half-band filter from that of a real half-band, transformation equations may be used. A real half-band filter with a frequency response

$$G(e^{j\omega}) = \begin{cases} 1, & 0\leqslant|\omega|<\frac{\pi}{2}, \\ 0, & \frac{\pi}{2}\leqslant|\omega|<\pi, \end{cases} \tag{13}$$

is related to $H(e^{j\omega})$, the complex half-band filter, according to

$$G(e^{j\omega}) = \tfrac{1}{2}H(e^{j(\omega+\pi)/2)}). \tag{14}$$

In the z domain, the relation between the transfer functions of a complex half-band filter $H(z)$ and a real half-band low-pass filter $G(z)$ is given by the transformation equation

$$H(z) = jG(-jz). \tag{15}$$

Substituting Eq. (3) in (15), the complex half-band filter derived from a real half-band IIR filter is thus

$$H(z) = A_0(-z^2) + jz^{-1}A_1(-z^2). \tag{16}$$

The realization of the complex half-band filter based on the above decomposition is shown in Fig. 3.

3. WDF equivalent

The relationship between a WDF, operating at a sampling rate $F = 1/T$, and its reference analogue filter is established in the frequency domain by the means of the bilinear transformation

$$s = \frac{2}{T}\left(\frac{z-1}{z+1}\right), \tag{17}$$

where s is the reference domain and z the digital domain. Direct realization of the digital filter obtained with Eq. (17) will produce signal flowgraphs with delay-free loops. To overcome this, WDFs use wave network characterization to obtain realizable digital structures.

The signals used in WDF representation are the wave quantities A representing the forward travelling waves and B representing the backward travelling waves. For a classical port characterized by a current I, a voltage V and a port resistance R, the equations that relate these parameters to the wave quantities are

$$A = V + IR, \tag{18a}$$

$$B = V - IR. \tag{18b}$$

Each port is thus represented by the wave quantities and a port resistance value. For a two-port network, the **S**-matrix describing the reflection and transmittance properties of the network is also used to describe WDFs with

$$\mathbf{B} = \mathbf{S}\mathbf{A}, \tag{19}$$

where

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}. \tag{20}$$

By using the wave characterization, each element in the analogue domain is represented as a two-port network. To form interconnection between elements, adaptors are used. These adaptors contain the multipliers and adders of the filter.

3.1. Adaptors

For an all-pass section, one corresponding WDF structure consists of a cascade of two-port adaptors. This structure corresponds, in the analogue domain, to a cascade of unit elements used in the design of microwave filters. The two-port adaptors used for the digital realization of the all-pass sections can have one or two multipliers. Realization with single-multiplier adaptors will give a canonic implementation of the digital filter. Fig. 4 shows three different configurations of a single-multiplier two-port adaptor.

Among the adaptors, the structure with its multiplier coefficient $\{\gamma, 1-\gamma, -(1+\gamma)\}$ that is less than half in absolute value should be selected due to its superior finite wordlength property [1]. These adaptors are equivalent since they implement the same relation as

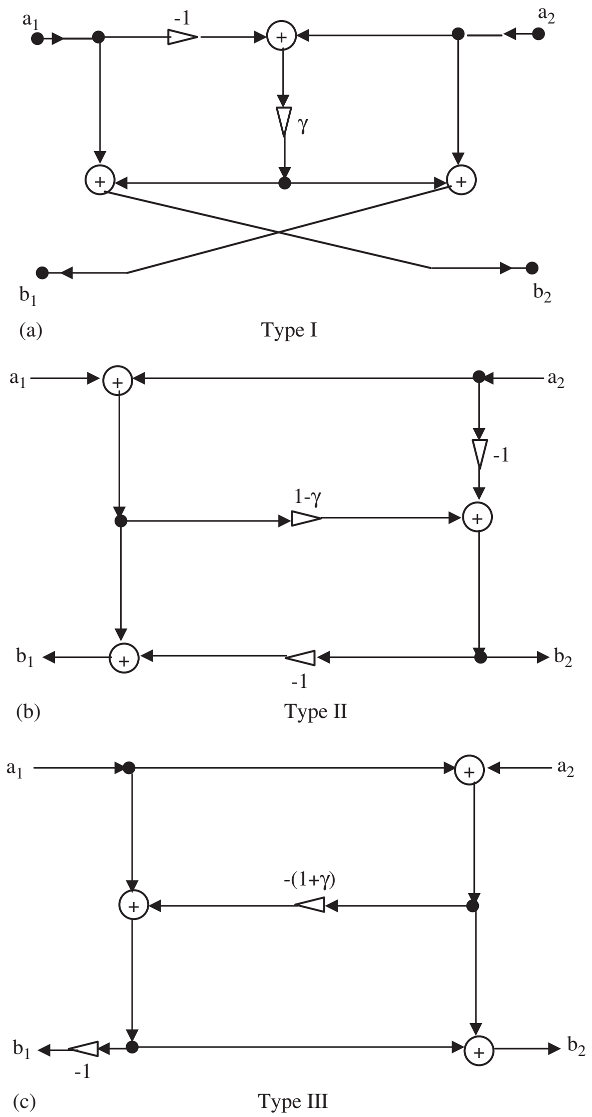

Fig. 4. Three equivalent two-port adaptors.

$$b_1 = \gamma a_1 + (1+\gamma)a_2, \tag{21a}$$

$$b_2 = (1-\gamma)a_1 - \gamma a_2. \tag{21b}$$

The multiplier coefficient, γ, is related to the port resistance R_1 and R_2 derived from the analogue reference filter according to

$$\gamma = \frac{R_1 - R_2}{R_1 + R_2}. \tag{22}$$

To obtain the coefficients of the adaptor from the all-pass digital domain transfer function, Schur

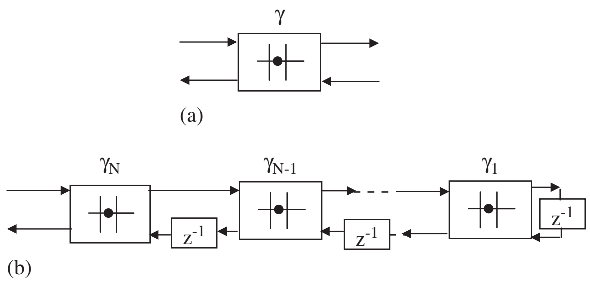

Fig. 5. (a) Two-port adaptor symbol, (b) WDF implementation of the all-pass structure.

parameterization can be used. One of the most important properties of the Schur algorithm is its orthonormality, which has been exploited to synthesize various types of lattice filters [14].

Let $A_N(z^{-1})$ be an all-pass function of order N, real, analytic inside the unit circle and such that

$$\left|A_N(z^{-1})\right| \leqslant 1, \quad \text{for} \quad \left|z^{-1}\right| = 1. \tag{23}$$

By defining

$$\gamma_N = A_N(0), \tag{24}$$

$A_{N-1}(z^{-1})$ is generated as

$$A_{N-1}(z^{-1}) = \frac{1}{z^{-1}} \frac{A_N(z^{-1}) - \gamma_N}{1 - \gamma_N A_N(z^{-1})}. \tag{25}$$

By repeating the Schur parameterization procedures in Eqs. (24) and (25), N parameters $\{\gamma_N, \gamma_{N-1}, \ldots, \gamma_1\}$ can be obtained representing the adaptor coefficients for the WDF. The symbol for a two-port adaptor is shown in Fig. 5(a) while Fig. 5(b) shows the interconnection of the adaptors obtained using Schur parameterization.

4. Design examples

To show the design process two examples are shown here, one for a real half-band WDF and the other for the corresponding complex half-band WDF. A fifth-order IIR digital filter satisfying Eq. (2) is selected having the following digital filter specifications:

Normalized passband edge frequency, ω_s: 0.4π.
Normalized stopband edge frequency, ω_p: 0.6π.
Stopband ripple, δ_s: 0.01550, ($R_s = -20 \log \delta_s = -36.193366$ dB).
Passband ripple, δ_p: 6.00661×10^{-5}, ($R_p = -20 \log(1-2\delta_p) = -1.043518 \times 10^{-3}$ dB).

The transfer function for this filter can be written in the form of Eq. (3) as

$$G(z) = \frac{1}{2}\left(\frac{0.23680414 + z^{-2}}{1 + 0.23680414 z^{-2}} + z^{-1}\frac{0.71490399 + z^{-2}}{1 + 0.71490399 z^{-2}}\right). \tag{26}$$

To obtain the WDF equivalent, successive Schur parameterization is applied to each of the two allpass sections. For the allpass section A_0, the resulting adaptor coefficients are {0, −0.23680414} and for A_1 they are {0, −0.71490399}. It can be observed that the magnitude of the non-zero coefficients can in fact be obtained directly from the transfer function.

The WDF configuration obtained by choosing the appropriate adaptor is as shown in Fig. 6. The adaptor as shown in Fig. 4(a) of Type I is required for the nonzero coefficient of section A_0 while that of Type III is required for section A_1. Furthermore, the zero coefficient adaptor can be omitted while maintaining the required number of delays. A minimum multiplier design for the WDF is obtained where only two single-multiplier adaptors are required.

The lowpass and highpass frequency responses for the minimum multiplier half-band WDF are shown in Figs. 7 and 8, respectively. Both the magnitude and phase responses are shown. The magnitude response meets the specifications required for the low-pass filter, while high-pass response is complementary to the low-pass response. Since the WDF is derived from an IIR

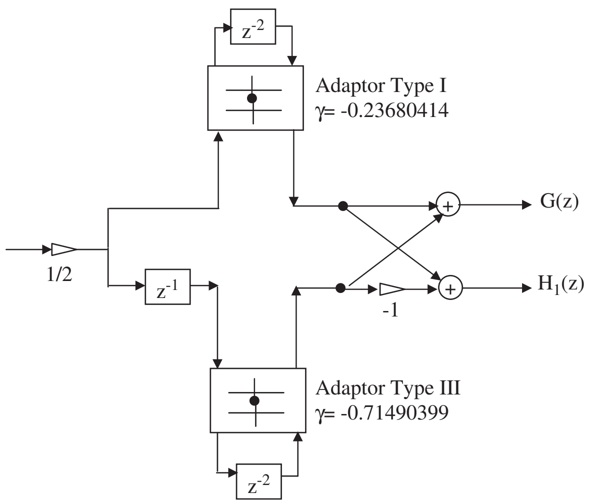

Fig. 6. Real Half-Band WDF.

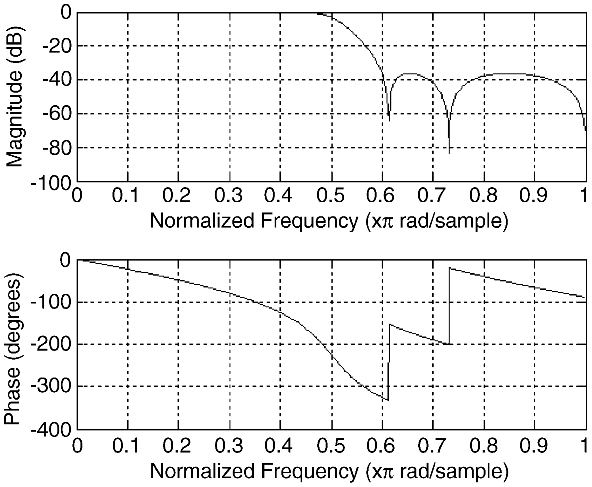

Fig. 7. Lowpass magnitude and phase responses for the half-band WDF.

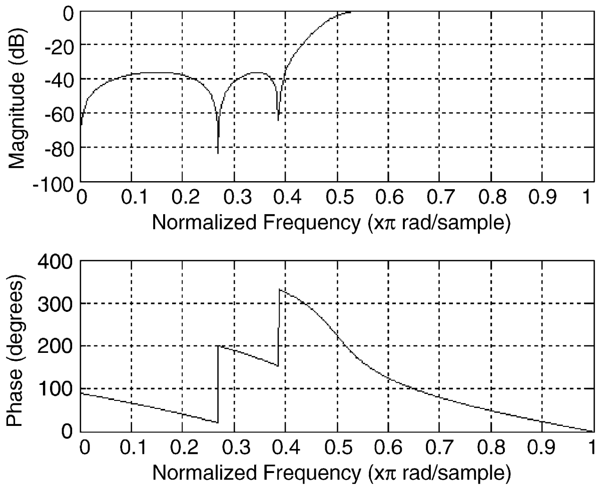

Fig. 8. Highpass magnitude and phase responses for the half-band WDF.

filter transfer function, strictly linear phase is not possible. However, the phase change is monotonically decreasing in the passband of both filters.

The equation for the corresponding complex half-band filter is formed by transforming Eq. (26) to the form of Eq. (15), obtaining the following:

$$H(z) = \frac{1}{2}\left(\frac{0.2368041466 - z^{-2}}{1 - 0.2368041466z^{-2}} + jz^{-1}\frac{0.7149039978 - z^{-2}}{1 - 0.7149039978z^{-2}}\right). \tag{27}$$

By applying Schur parameterization to the new allpass section A_0, the WDF multiplier adaptor coefficients obtained are {0, 0.23680414}, and for A_1 the coefficients are {0, 0.71490399}. The values are essentially equivalent to those obtained for the real-half band WDF except for the sign change. In the selecting the adaptors for the complex half-band WDF, the non-zero coefficient adaptor for A_0 requires a Type I as shown in Fig. 4(a), while the adaptor for A_1 requires a Type II adaptor as shown in Fig. 4(b). The resulting complex half-band WDF structure is shown in Fig. 9.

The magnitude response for the complex half-band filter is shown in Fig. 10. Except for frequencies near zero and π, the magnitude is unity throughout the passband. This is the main criterion for a complex half-band filter.

Fig. 11 shows the phase difference between the real, y_{re}, and imaginary, y_{im}, outputs. A 90° phase difference is obtained in the positive frequency ranges, 0 to π, while 270° phase difference is obtained for π to 2π. However, at frequencies near 0, π and 2π the phase response is not constant. In applying this minimum multiplier discrete Hilbert transformer realization, careful selection of the applied signal frequency with respect to the sampling frequency has to be observed so that it falls between the accepted ranges. However, it has the advantage of requiring only two multipliers for its implementation.

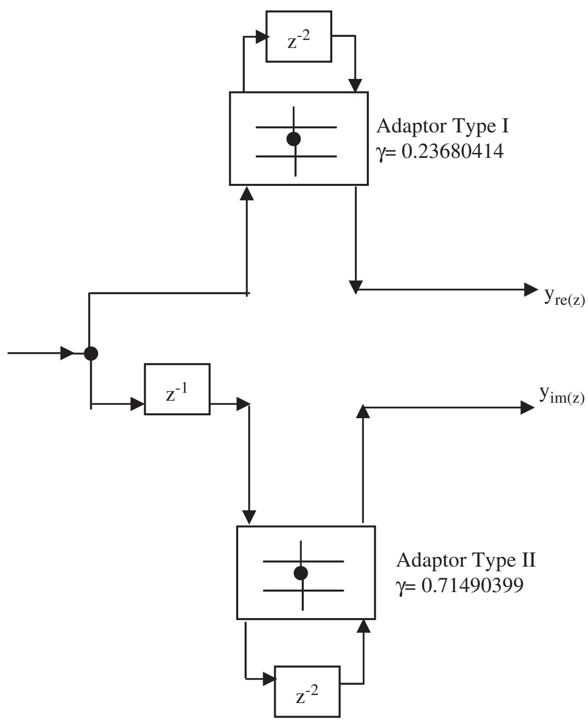

Fig. 9. Complex half-band WDF.

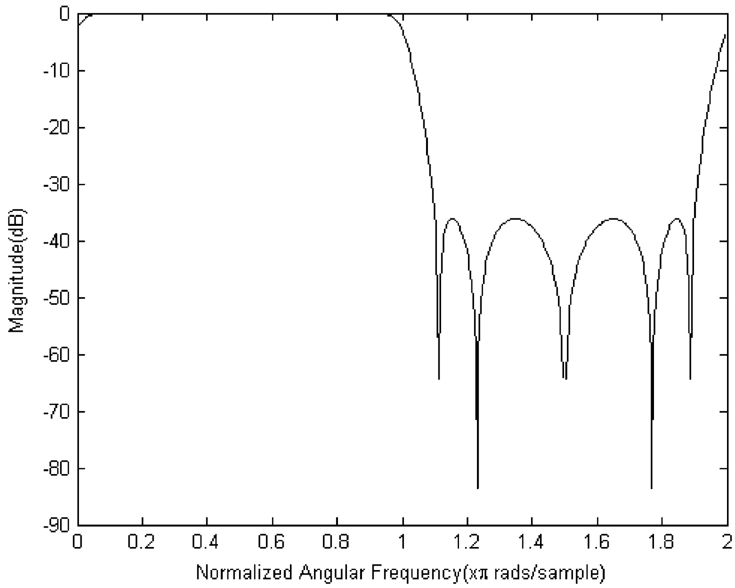

Fig. 10. Magnitude response of the complex half-band WDF.

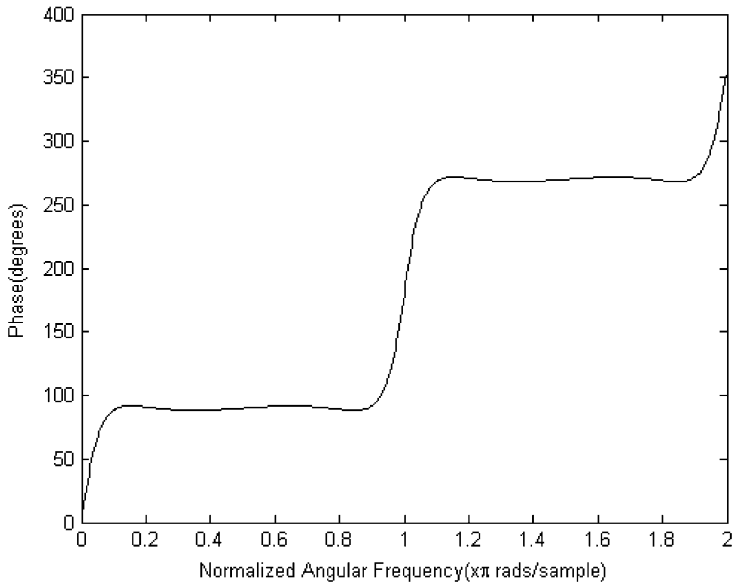

Fig. 11. Phase difference between real and imaginary components of the complex half-band WDF.

Conventionally, the Hilbert transformer is designed using FIR filters [5]. The performance of FIR filters designed using the popular Parks–McClellan algorithm [6] having the same specifications is shown in Fig. 12 where they are compared with the WDF. The plotted passband magnitude responses clearly show the superiority of the WDF in terms of filter order and hence the number of multiplier coefficients. The phase responses are not compared since FIR filters have linear phase while the WDF shown here have constant phase in a wide range of frequencies. In applications where linear phase is not required, the WDF filter is a better choice as it requires fewer coefficients to realize the same transfer function as an FIR filter.

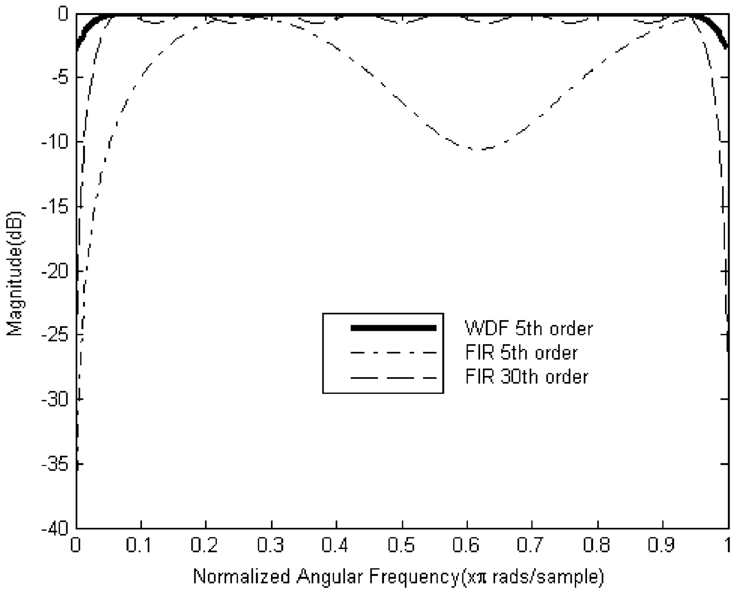

Fig. 12. Passband magnitude responses of WDF and conventional Hilbert transformer FIR filters.

5. Conclusion

This paper has shown the design of WDFs for discrete Hilbert transformer realization. The IIR filter transfer function is used directly in the extraction of the multiplier coefficients for the two-port adaptors. Examples are shown where the configuration of an adaptor is selected based on the value of the obtained multiplier coefficient. A minimum multiplier realization of a real half-band WDF is obtained to meet the low-pass digital filter magnitude specifications. The doubly complementary high-pass realization is simultaneously obtained from the same structure giving an efficient design. By applying a transformation equation, a complex half-band WDF suitable for the realization of the discrete Hilbert transformer with a minimum number of multiplier is obtained.

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