DESIGN OF HIGH-SPEED IIR FILTERS BASED ON ELLIPTIC MINIMAL Q-FACTORS PROTOTYPE

Miroslav D. Lutovac, *IRITEL Institute*, Beograd
Ljiljana D. Milić, *Mihajlo Pupin Institute*, Beograd

Abstract – *The total latency of the arithmetic operations in the critical direct loop of the second-order all-pass IIR section is reduced using a special class of the elliptic transfer function and thus the maximum sample frequency is increased. This is achieved by implementing one of two multipliers per section as a binary shifter, that is, without any hardware in FPGA or VLSI implementations. The second multiplier is implemented with a small number of shifters-and-adders using the quantization of the coefficients and optimization. We derive a new exact formula for those coefficients and we avoid all unnecessary computations in the optimization. New exact formulae provide efficient, very fast, simple and highly accurate computation suitable for standard software packages such as MATLAB.*

1. INTRODUCTION

Digital filters with high maximal sample frequency are important for high-speed applications and low power consumption. Stringent requirement on the stop-band attenuation implies very long coefficients. This results in a low maximal sample frequency. Recursive infinite impulse-response (IIR) filters can require a smaller number of arithmetic operations per sample than non-recursive finite impulse-response (FIR) filters. A drawback of using recursive filters is that the maximum sample frequency at which they can operate is bounded. The minimal value of the maximal sample frequency for a recursive algorithm is determined as a ratio of the number of delay elements in the direct loop to the total latency of the arithmetic operations in the critical direct loop of the *i*th-section [1]. Let us consider the all-pass implementation of IIR filters, which is shown to be convenient for high-speed and low-sensitivity design [1], [2], [3]. For a typical second-order all-pass section, that has two multipliers and four adders in the critical loop, the maximal sample frequency becomes

$$f_{\max} = \frac{1}{2T_{mult} + 4T_{add}} . \tag{1}$$

Replacing the multipliers with a small number of adders can increase the maximal sample frequency because the latency due to the multiplication is considerable larger than the latency due to an arithmetic operation of addition, $T_{mult} >> T_{add}$. When the value of the multiplier coefficient is a power-of-two, it can be implemented as a simple binary shift. By implementing the multipliers as binary shifters - that is without any hardware in FPGA or VLSI implementations, we remove the multipliers from the critical loop and the maximal sample frequency of the second-order IIR section reaches its maximal value.

Therefore, the values of the multiplication constants 0, ± ½, ± ¼, ... provide maximal sample frequency for the second order section. Since $T_{mult} >> T_{add}$, the simple combination of the powers-of-two for the values of the filter coefficients is also desirable for high-speed IIR filters.

In the number of publications [4-12], it was shown that an IIR filter derived by the bilinear transformation from an elliptic minimal Q factors (EMQF) prototype and realized as a parallel connection of two all-pass branches can attain high performances with a small number of shifters-and-adders in multipliers. Thus, this class of elliptic filters may be used in a high-speed IIR filter design.

In this paper, we derive the closed form relations for the design of high-speed IIR filters. The design is based on the special class of the odd-order elliptic IIR filters known as elliptic minimal Q-factors prototype filters (EMQF) [4].

2. ALL-PASS IMPLEMENTATION OF ODD-ORDER ELLIPTIC IIR FILTERS

The realization structures based on the parallel connection of two all-pass networks require half of the multipliers required for the direct canonical form [14-17]. This property is of significance for high-speed filtering. In this section, we review the basic definitions related to the all-pass implementation of elliptic IIR filters.

We can express the transfer function $H(z)$ of an odd-order elliptic IIR filter as a sum or difference of two all-pass functions $H_a(z)$ and $H_b(z)$:

$$H(z) = \frac{1}{2}\left(H_a(z) \pm H_b(z)\right). \tag{2}$$

where "+" stays for a low-pass filter and "-" for the complementary high-pass filter. The all-pass functions $H_a(z)$ and $H_b(z)$ are given by:

$$H_a(z) = z \prod_{1}^{[(n+3)/4]} \frac{\beta_i + \alpha_i(1+\beta_i)z^{-1} + z^{-2}}{1 + \alpha_i(1+\beta_i)z^{-1} + \beta_i z^{-2}} . \tag{3}$$

$$H_b(z) = \prod_{[(n+7)/4]}^{(n+1)/2} \frac{\beta_i + \alpha_i(1+\beta_i)z^{-1} + z^{-2}}{1 + \alpha_i(1+\beta_i)z^{-1} + \beta_i z^{-2}} . \tag{4}$$

where $[x]$ returns the integer value such that $x \le [x] < x+1$. With β=0, the first-order section, corresponding to the real pole of the transfer function, becomes:

$$z\frac{\alpha_1 z^{-1} + z^{-2}}{1+\alpha_1 z^{-1}} = \frac{\alpha_1 + z^{-1}}{1+\alpha_1 z^{-1}} . \tag{5}$$

The first-order and the second-order sections that may be used to implement (3-5) are described in [4-6], [10], [17], and [18].

3. HIGH-SPEED ELLIPTIC IIR FILTERS

The multipliers α_i and β_i (see equations (3) and (4)) are in the direct loop of the second order all-pass section. Our goal is to implement all coefficients α_i or all β_i as binary shifters (that is, their values to be 0, ± ½, ± ¼, ..., $\pm 1/2^m$). The coefficients β_i are different for each section because they are squared poles of the elliptic transfer function and they cannot be set by design to the required values. Therefore, we can search only for the coefficients α_i to be of the same values

$$\alpha_i = \alpha, \quad i = 2, 3, \ldots . \tag{6}$$

For a given specification, we can try to determine α to have a value from the preferred set of values α∈ {0, ±½, ±¼,...}. The special class of the odd-order elliptic IIR transfer functions (elliptic minimal Q-factors - EMQF [4]) has that property. All coefficients α_i, i>1, are of the same value. We can choose the value of α from a range of preferred values for a given specification [4]. For a chosen value of α, the transfer function is still strictly elliptic with equiripple property in the pass- and stop-band. The whole design margin can be used for the quantisation of the coefficients β_i and α_1. We optimize all coefficients β_i for minimal number of adders used for implementation of any β_i. The standard procedures for computing β_i can be time-consuming. Therefore we derive a new exact formula for β_i so that we avoid all unnecessary computations.

In the next section, we present the properties of EMQF filters that give a unique value of α for all the second order sections in the filter, and provide the efficient computation of the constants β_i.

4. COMPUTATION OF FILTER COEFFICIENTS

IIR filters derived by the bilinear transformation from the elliptic minimal Q factors (EMQF) prototypes are discussed in [4-7], [10]. It was shown that the z-plane poles of such an IIR filter are placed on the circle centered on the real axis and orthogonal with the unit circle.

The magnitude response of an EMQF filter is characterized by the equality of the ripple of the square magnitude response in the pass and stop bands [4]. Therefore, the filter order n and two independent parameters, instead of three in the case of a conventional elliptic filter, specify an EMQF filter. Let us take the filter order n and the pass- and stop-band edge frequencies f_p and f_s to determine an EMQF IIR filter. The frequency where the filter attenuation is 3 dB, f_{3dB}, is related with f_p and f_s by [4] (3 dB attenuation only approximately corresponds to the exact value 10 $\log_{10}(2)$):

$$\tan^2(\pi f_{3dB}) = \tan(\pi f_p)\tan(\pi f_s). \tag{7}$$

We define the filter selectivity factor ξ:

$$\xi = \frac{\tan(\pi f_s)}{\tan(\pi f_p)}. \tag{8}$$

If the filter order n of an EMQF filter is an odd number, there exist one real pole and $(n$-1)/2 complex conjugate pole pairs. Let us examine their computation.

(a) *Real pole, computation of* α_1

Using the property of the EMQF prototype, we obtain the following simple expression for real pole z_1:

$$z_1 = \frac{1-\tan(\pi f_{3dB})}{1+\tan(\pi f_{3dB})}. \tag{9}$$

and the coefficient of the first-order section, α_1:

$$\alpha_1 = -\frac{1-\tan(\pi f_{3dB})}{1+\tan(\pi f_{3dB})}. \tag{10}$$

(b) *Complex pole pairs, computation of* α *and* β_i

For the $(n$-1)/2 complex pole pairs, specified by $z_i = r_i e^{\pm j\theta_i}$, we form the second order polynomials $B_i(z^{-1})$:

$$B_i(z^{-1}) = 1 + \alpha(1+\beta_i)z^{-1} + \beta_i z^{-2}. \tag{11}$$

where

$$\alpha = -\cos(2\pi f_{3dB}), \quad \beta_i = |z_i|^2 = r_i^2. \tag{12}$$

This way the coefficient α is related directly with the specification, i.e. f_{3dB} frequency, and is a common constant for all the complex conjugate pole pairs of the filter. With slight modifications of filter boundary frequencies, we can adjust α to the convenient value. Adjusting the value of α to a simple combination of powers-of-two, we decrease the total latency in the critical loop of the second order sections and increase the maximal sample frequency of the filter.

The square pole magnitude β_i have to be computed for each pole pair. In this paper, we develop the closed form expression for the computation of β_i. We use the direct expression for the computation of the s-plane poles of an EMQF filter and develop the formula for the computation of β_i. For the pole s_i, we have [4, Eq. 10.23]:

$$s_i = \sqrt{\xi}\,\frac{-\sqrt{1-x_i^2}\sqrt{\xi^2 - x_i^2} + jx_i(\xi+1)}{\xi + x_i^2}. \tag{13}$$

where ξ is the selectivity factor, and x_i is the zero of the rational elliptic function. We use the Jacobi elliptic sine function to compute x_i [4]:

$$x_i = \mathrm{sn}\left(\left(\frac{2i-1}{n}+1\right)K_j\left(\frac{1}{\xi}\right), \frac{1}{\xi}\right). \tag{14}$$

where $K_j(1/\xi)$ is the complete elliptic integral of the first kind defined as

$$K_j\left(\frac{1}{\xi}\right) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-\left(\frac{1}{\xi}\right)^2 \sin^2\theta}}. \tag{15}$$

The z-plane pole z_i can be expressed using the bilinear transformation:

$$z_i = \frac{1+s_i\tan(\pi f_p)}{1-s_i\tan(\pi f_p)}. \tag{16}$$

To develop the expression for β_i, we start from the definition, equation (12). We represent β_i in terms of x_i, ξ, and f_p. We demonstrate in the Appendix that under the appropriate derivations, the following simple formula for β_i is reached:

$$\beta_i = \frac{\xi + x_i^2 - \lambda\sqrt{(1-x_i^2)(\xi^2 - x_i^2)}}{\xi + x_i^2 + \lambda\sqrt{(1-x_i^2)(\xi^2 - x_i^2)}}. \tag{17}$$

$$\lambda = \frac{2\sqrt{\xi}\tan(\pi f_p)}{\left(1+\xi\tan^2(\pi f_p)\right)}. \tag{18}$$

For a half-band filter, the selectivity factor is ξ=1/$\tan^2(\pi f_p)$ [4] giving λ =1 in (18).

5. APPLICATIONS

Using an EMQF transfer function, we can significantly increase the maximal sample frequency of an IIR filter by the proper selection of α. Selecting α as a power-of-two, we reduce the total latency in the critical loop of the second order section. The consequence is the limited number of discrete values of f_{3dB}. The density of f_{3dB} points is increased when implementing α as one addition and two shifts, i.e. when the value of α is selected from the set

$$\alpha \in \left\{\pm 1/2^m \pm 1/2^p, \pm\left(1 - 1/2^m \pm 1/2^p\right)\right\}$$

see [4] and [10]. This way, we replace one of two T_{mult} with T_{add} in equation (1), and thus still considerably increase f_{max}.

In an EMQF filter design, besides the filter order n, only two parameters can be independently specified. Usually, we select n, common constant α, and the stopband edge frequency f_s. The design steps are the following:

A. The filter order n is selected to fulfill the pass-band and stop-band specification [4].

B. The value of the common constant α is directly related to the specification and may be conveniently adjusted to the simple combination of powers-of-two, see equation (12) and references [4], [6], [10]. We compute f_{3dB} from the expression $f_{3dB} = \cos^{-1}(-\alpha)/(2\pi)$.

C. Compute the constant α_1 from (10).

D. Use n, α, and f_s to compute β_i:

D1. Compute ξ from equations (7) and (8).

D2. Use equations (14) and (15) to compute x_i. In MATLAB [19], the function ellipj can be used to determine the Jacobi elliptic function x=ellipj(((2*i-1)/n+1)*ellipke(1/ξ^2),1/ξ^2), and the function ellipke, that is ellipke(1/ξ^2), to determine the complete elliptic integral of the first kind, $K_j(1/\xi)$.

D3. Compute β_i from (17) and (18).

E. For a half-band filter, the selectivity factor is $\xi = 1/\tan^2(\pi f_p)$ [4], and in this case we obtain $\lambda = 1$. All coefficients α are reduced to zero. The first order section becomes a pure delay while all second order sections can be implemented using the first-order all-pass sections [14].

F. Using the selected n and ξ, the minimal stopband attenuation, A_s, and maximal pass-band ripple, A_p, can be computed from [4, p. 439].

The proposed procedure has been tested in MATLAB [19] for the various values of n, α, and f_s. For example, for $\alpha = -1/2$, $f_s = 0.2$, and $n = 3, 5, \ldots, 31$, an equiripple characteristic in the stop-band is obtained up to $n = 27$ that gives the minimum stop-band attenuation of $A_s = 196$ dB.

The proposed computation procedure is simple, highly accurate and very fast. Alternatively, one can use an existing routine for elliptic filters to design an EMQF filter such as ellip in MATLAB. This alternative approach has several disadvantages in comparison with the design method presented in this paper:

a) Before applying a conventional program for elliptic filters, the input parameters (value of the pass-band and stop-band ripple, filter order n and the selectivity factor ξ) have to be settled in accordance, see [4, p. 439-440].

b) A routine has to be written to establish the relation between the common constant α and the filter specification.

c) The accuracy of the computation becomes critical in some cases. The MATLAB function ellip works well up to the filter order n=11 with the stopband attenuation A_s = 76.38 dB

High-speed filters described in this paper can be implemented as a parallel combination of all-pass networks using all-pass sections from [4], [5], [6], [10], [17], and [18]. In this paper, we demonstrate how the total latency in the second order section can be decrease by a proper selection of one of two multiplication constants. Since with the adjustment of the constant α, we keep the strictly elliptic characteristic of the filter, the whole filter margin is left for the quantization of the remaining coefficients β_i. The procedure for the quantization of β_i that is described in [10] can bring the additional benefits in increasing the maximal sample frequency. This quantization procedure is simplified by using expressions (17) and (18) for computation of β_i.

6. CONCLUSION

We present a new approach in designing high-speed IIR digital fitters. The contributions of the paper are the following:

(a) We show that using the elliptic minimal Q factors (EMQF) transfer function and implementation based on parallel connection of two all-pass branches, the maximal sample frequency of an elliptic IIR filter can be significantly increased.

(b) We develop the closed form expressions for the computation of the coefficients in first-order and second-order all-pass sections of filter branches.

(c) We present a simple and highly accurate computation procedure suitable for standard software packages such as MATLAB.

The design approach presented in the paper is direct and therefore very fast. The proposed method is simple to use since the values of the coefficients are related with the filter specification.

REFERENCES

[1] H. Johansson and L. Wanhammar, "Design and Implementation of Multirate Digital Filters," in G. Jovanovic Dolecek ed., *Multirate Systems: Design and Application*, Idea Group Publishing, pp. 257-292, 2002.

[2] 2. H. Johansson and L. Wanhammar, "High-speed recursive digital filters based on the frequency-response masking approach", *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 47, no. 1, pp. 48-61, 2000.

[3] J. Y. Kaakinen and T. Saramaki, "Design of very low-sensitivity and low-noise recursive filters using a cascade of low-order lattice wave digital filters," *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 45, no. 7, pp. 906-914, 1999.

[4] M. D. Lutovac, D. V. Tošić, B. L. Evans, *Filter Design for Signal Processing Using MATLAB and Mathematica*, New Jersey, Prentice Hall, 2001.

[5] M. D. Lutovac and Lj. D. Milić, "Lattice wave digital filters with a reduced number of multipliers," Yugoslav IEEE MTT Chapter Inform., vol. 2, no. 3, pp. 29-39, June 1996.

[6] M. D. Lutovac and Lj. D. Milić, "Design of computationally efficient elliptic IIR filters with a reduced number of shift-and-add operations in multipliers," *IEEE Trans. Signal Processing*, vol. 45, no. 10, pp. 2422-2430, Oct. 1997.

[7] Lj. D. Milić and M. D. Lutovac, "Low-complexity lattice wave digital filters," *Proc. 13th European Conf. Circuit Theory Design, ECCTD'97*, Budapest, Hungary, Aug. 1997, pp. 1141-1146.

[8] Lj. D. Milić and M. D. Lutovac, "Design of multiplierless IIR filters," *IEEE Int. Conf. Acoustics, Speech, Signal Processing, ICASSP97*, Munich, Germany, Apr. 1997, pp. 2201-2204.

[9] M. D. Lutovac and Lj. D. Milić, "Design of multiplierless elliptic IIR halfband filters and Hilbert transformers," *European Signal Processing Conference, EUSIPCO'98*, Rodos, Greece, Sept. 1998, pp. 291-294.

[10] Lj. D. Milić and M. D. Lutovac, "Design of multiplierless elliptic IIR filters with a small quantization error," *IEEE Trans. Signal Processing*, Vol.47, No. 2, pp. 469-479, February 1999.

[11] M. D. Lutovac and Lj. D. Milić, “Short wordlength selective IIR filters,” *Proceedings of European Conference on Circuit Theory and Design, ECCTD’99*, pp. 277-280, Stresa, Italy 1999.

[12] M. D. Lutovac and Lj. D. Milić, “Approximate linear phase multiplierless IIR halfband filter”, *IEEE Signal Processing Letters*, Vol. 7, No. 3 pp. 52-53, March 2000.

[13] Lj. Milić and M. Lutovac, “Efficient Multirate Filtering,” in G. Jovanovic - Dolecek ed. *Multirate Systems: Design and Applications*, Idea Group Publishers, 2002.

[14] Fetweis, H. Levin, and A. Sedlemeyer, “Wave digital lattice filters,” *Int. J. Circuit Theory Appl.*, pp. 203-211, June 1974.

[15] S. K. Mitra, “Digital all-pass networks,” *IEEE Trans. Circuits Syst.*, vol. CAS-21, pp. 688-700, Sept. 1974.

[16] P.P. Vaidyanthan, S.K. Mitra, and Y.Neuvo, “A new approach to the realization of low-sensitivity IIR digital filters,” *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 350-361, Apr. 1986.

[17] R. Ansari and B. Liu, “A class of low-noise computationally efficient recursive digital filters with application to sampling rate alternations,” *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-33, pp. 90-97, Feb. 1985.

[18] L. Gaszi, “explicit formulas for lattice wave digital filters,” *IEEE Trans. Circuit Syst.*, vol. CAS-32, pp. 68-88, Jan. 1985.

[19] The MathWorks, Inc., *Using MATLAB*, Version 6.1 Release 12.1, Natick, MA 01760-2098, 2001.

APPENDIX

In this appendix, we present the proof for the expressions (17) and (18). We start from the expression for z_i given in (12) and replace s_i with the expression (13). After multiplying numerator and denominator with $(\xi+x_i^2)$, we obtain for z_i:

$$z_i = \frac{(\xi + x_i^2) - \sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)} + i x_i(\xi+1)\sqrt{\xi}\,\tan(\pi f_p)}{(\xi + x_i^2) + \sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)} - i x_i(\xi+1)\sqrt{\xi}\,\tan(\pi f_p)}$$

(A.1)

We apply (12) and (A.1) to express β_i in terms of x_i, ξ, and f_p:

$$\beta_i = \frac{\left((\xi + x_i^2) - \sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)}\right)^2 + \left(x_i(\xi+1)\sqrt{\xi}\,\tan(\pi f_p)\right)^2}{\left((\xi + x_i^2) + \sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)}\right)^2 + \left(x_i(\xi+1)\sqrt{\xi}\,\tan(\pi f_p)\right)^2}$$

(A.2)

Let us simplify the first term in the numerator:

$$\left((\xi + x_i^2) - \sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)}\right)^2 =$$

$$(\xi + x_i^2)^2 + \xi\,\tan(\pi f_p)^2(1-x_i^2)(\xi^2-x_i^2)$$

$$-2(\xi + x_i^2)\sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)} \qquad \text{(A.3)}$$

By adding the second term of the numerator, $x_i^2(\xi+1)^2\xi\tan(\pi f_p)^2$, to equation (A.3) and by putting together all the factors associated with the term $\xi\tan(\pi f_p)^2$, the following expression is obtained for the numerator:

$$(\xi + x_i^2)^2 + \left((1-x_i^2)(\xi^2-x_i^2) + x_i^2(\xi+1)^2\right)\xi\,\tan(\pi f_p)^2$$

$$-2(\xi + x_i^2)\sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)} \qquad \text{(A.4)}$$

Factor associated to $\xi\tan(\pi f_p)^2$ becomes:

$$\left((1-x_i^2)(\xi^2-x_i^2) + x_i^2(\xi+1)^2\right) = (\xi + x_i^2)^2 \qquad \text{(A.5)}$$

The numerator expressed by equation (A.4), in which the factor associated to $\xi\tan(\pi f_p)^2$ is simplified using equation (A.5), becomes:

$$(\xi + x_i^2)^2 + (\xi + x_i^2)^2\xi\,\tan(\pi f_p)^2$$

$$-2(\xi + x_i^2)\sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)} \qquad \text{(A.6)}$$

By putting together the first two terms associated with the term $(\xi + x_i^2)^2$, the numerator simplifies to:

$$(\xi + x_i^2)^2\left(1 + \xi\,\tan(\pi f_p)^2\right)$$

$$-2(\xi + x_i^2)\sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)} \qquad \text{(A.7)}$$

Using the similarity of the expressions in the nominator and denominator, and the same procedure for simplifying the numerator, the denominator becomes

$$(\xi + x_i^2)^2\left(1 + \xi\,\tan(\pi f_p)^2\right) +$$

$$2(\xi + x_i^2)\sqrt{\xi}\,\tan(\pi f_p)\sqrt{(1-x_i^2)(\xi^2-x_i^2)} \qquad \text{(A.8)}$$

After dividing the numerator and the denominator with the factors $(\xi+x_i^2)$ and $(1+\xi+\tan(\pi f_p)^2)$, we obtain the final expression for β_i, see equations (17) and (18).

Sadržaj – Ukupno kašnjenje aritmetičkih operacija u kritičnoj direktnoj petlji svepropusne sekcije drugog reda smanjene su korišćenjem specijalne klase eliptičkih funkcija a time je povećana maksimalna učestanost odabiranja. Ovo je ostvareno realizacijom jednog od dva množača sekcije kao binarni pomerač što ne zahteva hardver u FPGA ili VLSI implementacijama. Drugi množač je realizovan sa malim brojem pomerača i sabirača korišćenjem kvantizacije koeficijenata i optimizacionih postupaka. Izvedena je nova formula za koeficijenate funkcije prenosa i time su izbegnuta nepotrebna izračunavanja u optimizaciji. Nove formule omogućavaju brzo, jednostavno i precizno izračunavanje koeficijenata filtra korišćenjem standardnih softverskih paketa kao što je MATLAB.

PROJEKTOVANJE FILTARA ZA VELIKE BRZINE OBRADE NA OSNOVU PROTOTIPA ELIPTIČKIH IIR FILTARA SA MINIMALNIM Q FAKTORIMA

Miroslav D. Lutovac i Ljiljana D. Milić