

Two-Channel IIR/FIR Filter Banks with Very Low-Complexity Analysis or Synthesis Filters: Finite Wordlength Effects

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Abstract

A new class of two-channel IIR/FIR filter banks was introduced in [1] with half-band IIR analysis filters and FIR synthesis filters. This type of filter bank features very low-complexity analysis filters and simultaneously a low overall complexity. In this paper, we consider finite-wordlength effects of these filter banks.

1 Introduction

A new two-channel IIR/FIR filter bank class was recently proposed in [1]. In this filter bank, the analysis filters are half-band IIR filters whereas the synthesis filters are FIR filters. The complexity of the analysis filter bank is therefore very low whereas that of the synthesis filter bank is higher. The overall complexity is however comparable to, or even lower than, that of conventional filter banks. The proposed filter banks are of interest in applications where it is important to have a very low complexity in the analysis part, and where one can afford a higher complexity in the synthesis part. One example is hybrid discrete-time/digital filter banks for analog-to-digital conversion, where the analysis filters are discrete-time filters, such as switched capacitor-filters. Since it is more difficult to implement high-resolution discrete-time filters than digital filters, it is essential to use low-order analysis filters. In this paper we examine the finite-wordlength effects of the filter banks proposed in [1].

2 Review of the two-channel IIR/FIR filter banks introduced in [1]

The proposed filter banks in [1] belong to the general two-channel maximally decimated filter bank shown in Fig. 1. The z -transforms of the output $y(n)$ and input $x(n)$ are related according to

$$Y(z) = V_0(z)X(z) + V_1(z)X(-z) \quad (1)$$

with

$$\begin{aligned} V_0(z) &= \frac{H_0(z)G_0(z) + H_1(z)G_1(z)}{2} \\ V_1(z) &= \frac{H_0(-z)G_0(z) + H_1(-z)G_1(z)}{2} \end{aligned} \quad (2)$$

where $V_0(z)$ and $V_1(z)$ are the distortion and aliasing transfer functions, respectively.

The analysis filters are half-band IIR lowpass and highpass filters, respectively. Their transfer functions can be written in polyphase form as

$$\begin{aligned} H_0(z) &= 0.5[A_0(z^2) + z^{-1}A_1(z^2)] \\ H_1(z) &= 0.5[A_0(z^2) - z^{-1}A_1(z^2)] \end{aligned} \quad (3)$$

where $A_0(z)$ and $A_1(z)$ are real stable allpass filters and the

order of $H_0(z)$ and $H_1(z)$ is odd. A lowpass and highpass Cauer (elliptic) filter pair falls into this class of analysis filters yielding very low-complexity analysis filters. The filters $H_0(z)$ and $H_1(z)$ can be expressed as

$$H_0(z) = \frac{N(z)}{D(z^2)}, \quad H_1(z) = \frac{N_c(z)}{D(z^2)} \quad (4)$$

where

$$\begin{aligned} N(z) &= \frac{1}{2}[z^{-2K_0}D_0(z^{-2})D_1(z^2) \\ &\quad + z^{-1}z^{-2K_1}D_0(z^2)D_1(z^{-2})] \\ N_c(z) &= \frac{1}{2}[z^{-2K_0}D_0(z^{-2})D_1(z^2) \\ &\quad - z^{-1}z^{-2K_1}D_0(z^2)D_1(z^{-2})] \\ D(z^2) &= D_0(z^2)D_1(z^2) \end{aligned} \quad (5)$$

with

$$A_i(z) = \frac{z^{-K_i}D_i(z^{-1})}{D_i(z)} \quad (6)$$

where K_i denotes the order of $A_i(z)$ and $D_i(z)$.

The transfer functions of the synthesis filters are

$$\begin{aligned} G_0(z) &= 2F_0(z^2)F_1(z)F_2(z^2) \\ G_1(z) &= 2F_0(z^2)F_{1c}(z)F_2(z^2) \end{aligned} \quad (7)$$

where

$$F_0(z) = D(z), \quad F_1(z) = N(z), \quad F_{1c}(z) = -N_c(z) \quad (8)$$

Here, $F_0(z)$ is a nonlinear-phase FIR filter whereas $F_1(z)$, and $F_{1c}(z)$ are odd-order linear-phase FIR filters with symmetric and anti-symmetric impulse responses respectively. Further, $F_2(z)$ is an even-order linear-phase filter with a symmetric impulse response. (In fact, the order of $F_2(z)$ must be $4L$ for some integer L .)

The distortion and the aliasing transfer functions are easily derived as

$$V_0(z) = z^{-K}D(z^2)D(z^{-2})F_2(z^2), \quad V_1(z) = 0 \quad (9)$$

Obviously $V_0(z)$ has a linear phase, provided that $F_2(z)$ is a linear-phase filter, since a transfer function of the form $H(z)H(z^{-1})$ is real for $z = e^{j\omega T}$. Further, the aliasing function is exactly zero.

By making use of polyphase decimator and interpolator structures, the analysis and synthesis banks can be efficiently realized as shown in Figs. 2. and 3, respectively. Here, $F_{10}(z)$ and $F_{11}(z)$ are the polyphase components of $F_1(z)$ and $F_{1c}(z)$ according to

$$\begin{aligned} F_1(z) &= F_{10}(z^2) + z^{-1}F_{11}(z^2) \\ F_{1c}(z) &= F_{10}(z^2) - z^{-1}F_{11}(z^2) \end{aligned} \quad (10)$$

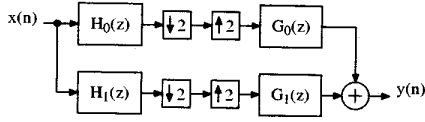


Figure 1. Two-channel digital filter bank.

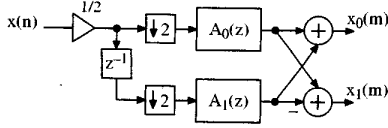


Figure 2. Realization of the analysis filter bank.

To obtain the structure in Fig. 3, we have also utilized the fact that, placing $F_0(z^2)$ and $F_2(z^2)$ after upsampling by two, is equivalent to placing $F_0(z)$ and $F_2(z)$ before upsampling by two. It should be noted that the structure in Fig. 3 is merely one option. Obviously, there exist several other alternatives. Whichever structure one chooses, $F_2(z)$ should however be placed in the middle in order to minimize the roundoff noise as will become clear in Section 4.

3 Structures achieving a linear-phase distortion function and zero aliasing with quantized coefficients

This section shows how to achieve zero aliasing and a linear-phase distortion function under finite-arithmetic conditions*. In practice, one can normally allow small aliasing errors and deviations from an exact linear phase. The structure in this section is therefore not necessarily the best one from an implementation point of view. It has the advantage though that it makes the search for optimal quantized coefficients easier.

The allpass filters $A_0(z)$ and $A_1(z)$ can be realized by cascading a number of first-order allpass sections. Thus, their transfer functions can be written as in (6) with

$$D_i(z) = \prod_{k=0}^{K_i} (1 - \alpha_{ik} z^{-1}) \quad (11)$$

The first-order allpass sections can, e.g., be realized using wave digital filters which makes it possible to obtain robust filters under finite-arithmetic conditions [2]. The overall analysis filters belong in this case to the well known bireciprocal lattice WDFs [2]. One alternative first-order section is shown in Fig. 4.

Zero aliasing and a linear-phase distortion function, when the filter coefficients are quantized, are now obtained by realizing $F_0(z)$ and the polyphase components $F_{10}(z)$, and $F_{11}(z)$ in cascade form according to

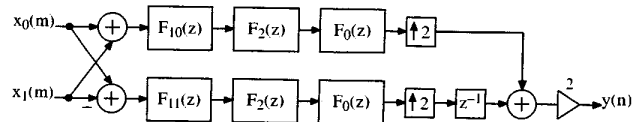


Figure 3. Realization of the synthesis filter bank.

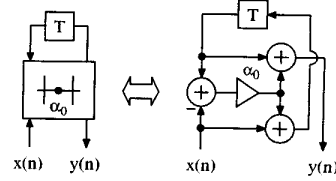


Figure 4. First-order wave digital allpass filter section.

$$F_0(z) = \prod_{k=1}^{K_0} (1 - \alpha_{0k} z^{-1}) \prod_{k=1}^{K_1} (1 - \alpha_{1k} z^{-1})$$

$$F_{10}(z) = \frac{1}{2} \prod_{k=1}^{K_0} (-\alpha_{0k} + z^{-1}) \prod_{k=1}^{K_1} (1 - \alpha_{1k} z^{-1}) \quad (12)$$

$$F_{11}(z) = \frac{1}{2} \prod_{k=1}^{K_0} (1 - \alpha_{0k} z^{-1}) \prod_{k=1}^{K_1} (-\alpha_{1k} + z^{-1})$$

Apparently, with $D_0(z)$ and $D_1(z)$ as in (11), and $F_0(z)$, $F_{10}(z)$, and $F_{11}(z)$ as in (12), (8) is satisfied which means that (9) holds.

3.1 Design of filters with quantized coefficients

In order to minimize the complexity of the analysis bank, the first step is to design the analysis filters to meet some prescribed criteria on their magnitude responses. For most practical specifications, the orders of $H_0(z)$ and $H_1(z)$ are very low (since they can be Caue filters when using infinite precision coefficients). Thus, their corresponding structures contain very few filter coefficients. It is therefore easy to design the filters with quantized coefficients by simply doing an exhaustive search. To speed up the search, the search space can be reduced by employing the method in [3]. With the analysis filters fixed, the next step is to design the synthesis filters so that the magnitude response of the distortion transfer function is as close to unity as desired. The filters $F_0(z)$, $F_1(z)$, and $F_{1c}(z)$ are given by (12). The remaining filter $F_2(z)$ can then be designed using, e.g., mixed integer linear programming.

4 Roundoff noise

In this section, we consider the round-off noise of the filter banks. The noise propagation through the synthesis bank is determined by the lowpass and highpass frequency responses, not the filter structures. Therefore, we will only consider how the noise generated within the analysis and synthesis filters affect the respective outputs.

All filters are scaled using the L_2 -norm. We assume that all inputs to the multipliers are scaled (which is the common way to scale filters using two's-complement arithmetic [4]). We further assume that rounding takes place after each multiplier and that the errors can be modeled as uncorrelated white noise

*It is possible to achieve exact zero-aliasing under finite-arithmetic conditions with other structures as well, like single direct form FIR filter structures, but then the coefficient wordlengths need to be very long.

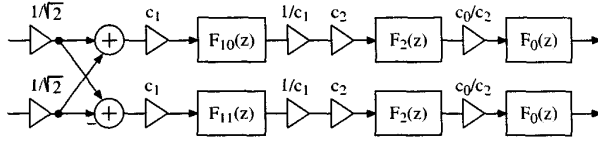


Figure 5. Scaling of the synthesis bank.

sources having zero mean and variance σ^2 .

Consider first the analysis bank. This bank uses well known filters (allpass WDFs) for which scaling and noise propagation has been treated earlier, see e.g. [5]. Therefore, we directly give the noise variance at the output of the analysis filter. Using scaled first-order allpass sections according to Fig. 4, the output variance (in both branches) becomes

$$\sigma_{x_0}^2 = \sigma_{x_1}^2 = 2 \sum_{k=1}^{K_0} \frac{1}{1-\alpha_{0k}^2} + 2 \sum_{k=1}^{K_1} \frac{1}{1-\alpha_{1k}^2} \quad (13)$$

Consider next the synthesis bank. For the sake of simplicity we treat here the cases in which the filters $F_{10}(z)$, $F_{11}(z)$, and $F_2(z)$, and $F_0(z)$ are realized using direct-form FIR filter structures. One possible realization of the synthesis bank was shown earlier in Fig. 1. Apparently, for each branch there are six different ways of ordering the subfilters. In total there are thus 36 possible combinations. However, as far as the noise variance is concerned, when using L_2 -norm scaling, the upper and lower branches are equivalent because the L_2 -norms of $F_{10}(z)$ and $F_{11}(z)$ are equal. It thus suffices to investigate the six options in one branch.

The filter bank is in principle scaled as illustrated in Fig. 5. In practice, the scaling constants are combined with the multiplier constants in the direct-form structures. Under the above assumptions, it is readily confirmed that the noise variance at the output of the upper branch in Fig. 5 is

$$\sigma_y^2 = \frac{\|F_{11}\|_2^2 \cdot \|F_0 F_2\|_2^2}{\|F_0 F_1 F_2\|_2^2} \sigma_1^2 + \frac{\|F_{10}\|_2^2 \cdot \|F_1 F_2\|_2^2}{\|F_0 F_1 F_2\|_2^2} \sigma_2^2 + \sigma_0^2 \quad (14)$$

where $\|H\|_2$ denotes the L_2 -norm of $H(z)$ and is given by

$$\|H\|_2^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |H(e^{j\omega T})|^2 d\omega T = \sum_{n=-\infty}^{\infty} |h(n)|^2 \quad (15)$$

Further,

$$\sigma_i^2 = N_i \sigma^2 \quad (16)$$

for $i = 0, 1, 2$, where N_i denotes the number of quantizations within $F_i(z)$. The other five combinations yield expressions similar to that in (14).

We consider the following six different specifications for the analysis filters:

- Spec. 1: $\omega_c T = 0.4\pi$, $\omega_s T = 0.6\pi$, $\delta_s = 0.0154195$
- Spec. 2: $\omega_c T = 0.45\pi$, $\omega_s T = 0.55\pi$, $\delta_s = 0.0441053$
- Spec. 3: $\omega_c T = 0.4\pi$, $\omega_s T = 0.6\pi$, $\delta_s = 0.0022026$
- Spec. 4: $\omega_c T = 0.45\pi$, $\omega_s T = 0.55\pi$, $\delta_s = 0.0096037$
- Spec. 5: $\omega_c T = 0.4\pi$, $\omega_s T = 0.6\pi$, $\delta_s = 0.0003761$
- Spec. 6: $\omega_c T = 0.45\pi$, $\omega_s T = 0.55\pi$, $\delta_s = 0.0020894$

In all six cases the distortion function is less than 0.05 in magnitude.

Table 1 summarizes the complexities and delays in the different cases. As a reference, we also consider QMF FIR filter banks with magnitude distortion that uses odd-order linear-phase analysis and synthesis filters of equal orders [6], [7]. We see that the new filter banks have a very low-complexity analysis filter bank and simultaneously a low overall complexity.

Table 2 summarizes the number of extra bits ΔW needed* in the analysis filters in the upper branch as well as the lower branch (since they are equal).

Table 3 summarizes the number of extra bits needed in the synthesis filters. It can be seen that in all six cases, the noise variance is minimized by placing $F_2(z)$ in the middle.

Table 4 summarizes the corresponding number of extra bits required for the FIR filter banks. Compared with the FIR banks, the number of extra bits required in the analysis filters of the presented filter banks is somewhat smaller. For the synthesis filters the proposed filter banks require between 1.5 and 5.1 more bits, for the different cases, than the FIR filter bank, if $F_2(z)$ is placed in the middle

Table 1. Comparison between the new and a class of QMF FIR filter banks.

	Number of multiplications per input/output sample			Delay in samples
	Analysis	Synthesis	Overall	
New, Spec. 1	1	18	19	27
FIR, Spec. 1	12	12	24	23
New, Spec. 2	1	24	25	41
FIR, Spec. 2	17	17	34	33
New, Spec. 3	1.5	26	27.5	44
FIR, Spec. 3	16	16	32	31
New, Spec. 4	1.5	36	37.5	64
FIR, Spec. 4	25	25	50	49
New, Spec. 5	2	35	37	61
FIR, Spec. 5	21	21	42	41
New, Spec. 6	2	48	50	87
FIR, Spec. 6	32	32	64	63

*Here, ΔW is the number of extra bits required to obtain the same noise variance as in the case where only one quantization takes place at the output whereas infinite precision arithmetic is used within the filters. This measure makes it possible to directly compare different structures.

Table 2. Results for the analysis bank in the proposed system.

ΔW					
Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5	Spec. 6
1.3	1.5	1.6	1.8	1.9	2.1

Table 3. Results for the synthesis bank in the proposed system.

Ordering of filters	ΔW					
	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5	Spec. 6
0, 1, 2	5.2	6.7	8.0	10.2	11.1	13.9
0, 2, 1	3.3	4.1	4.7	5.8	6.2	7.6
1, 0, 2	4.9	6.4	7.8	10.0	10.9	13.7
1, 2, 0	3.4	4.1	4.7	5.9	6.2	7.6
2, 0, 1	5.7	7.5	8.6	11.1	11.8	14.8
2, 1, 0	5.7	7.5	8.6	11.1	11.8	14.8

Table 4. Results for QMF FIR filter banks.

QMF	ΔW					
	Spec. 1	Spec. 2	Spec. 3	Spec. 4	Spec. 5	Spec. 6
Analysis	2.3	2.5	2.5	2.8	2.7	3.0
Synthesis	1.8	2.0	2.0	2.3	2.2	2.5

5 Coefficient sensitivity

To assess the coefficient sensitivity we make use of Monte Carlo analysis. Naturally, as opposed to analog systems, the multiplier coefficients in an implementation of a digital system are not varied once they have been fixed. However, a Monte Carlo analysis is still useful in the digital case when the sensitivity of a structure is investigated.

The analysis bank uses well known filters (allpass WDFs) for which sensitivity has been treated earlier [5]. Therefore, we will only consider the sensitivity of the synthesis filters. We assume that the analysis filters have fixed constants and that $F_0(z)$ and $F_1(z)$ are selected as outlined in Section 4. It thus remains to consider the sensitivity of $F_2(z)$. Figure 6 shows the Monte Carlo analysis of $V_0(z)$ for Spec. 1 using a coefficient wordlength in $F_2(z)$ of 13 bits (including sign, integer, and fractional bits). We see that the errors are small around $\pi/2$ and large for low and high frequencies. We have observed this for the other cases as well. As a reference we again use QMF FIR filter banks of the type considered in Section 4. Figure 7 shows the Monte Carlo analysis of $V_0(z)$ for Spec. 1 using a coefficient wordlength of 11 bits. Our filter bank thus needs about two more bits. For the six specifications in Section IV our banks roughly need between 2 and 7 extra coefficient bits.

6 Conclusion

A new class of two-channel IIR/FIR filter banks was recently introduced in [1]. This type of filter bank features very low-complexity analysis filters and simultaneously a low overall complexity. This paper considered finite-wordlength effects of these filter banks. We showed how to achieve zero aliasing and a linear-phase distortion function under finite-arithmetic

conditions. Further, we provided round-off noise and sensitivity analysis and compared the new filter banks with a class of FIR filter banks.

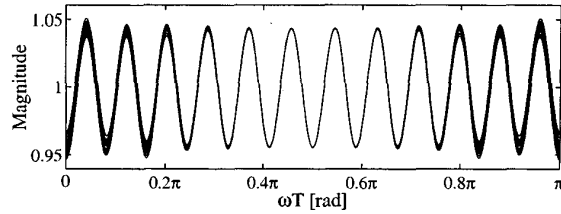


Figure 6. Monte carlo analysis of the proposed synthesis filters.

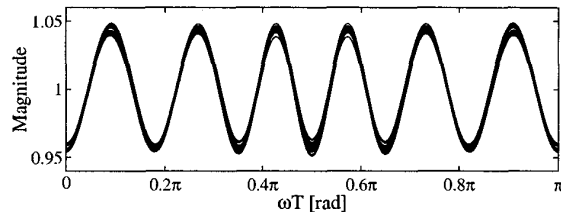


Figure 7. Monte carlo analysis of the QMF FIR.

The analysis showed that the synthesis filters in the new banks typically need a few bits more for representing data as well as filter coefficients. The main motivation for introducing the new banks is however that we want to have a very low complexity in the analysis part whereas we allow a higher complexity in the synthesis part. Taking the number of arithmetic operations required as well as finite wordlength effects into consideration, the results in this paper show that the overall complexity of the new banks are comparable to more conventional banks. At the same time, the analysis filters have a very low complexity. One should however keep in mind that it is difficult to make comparisons between different filter bank classes since they have different properties. For example, the IIR analysis filters in the new banks have a nonlinear-phase whereas the FIR analysis filters in the filter banks used for comparison have a linear phase. Nevertheless, our results show that the proposed filter banks are attractive when it is desired to have a low complexity analysis bank and nonlinear-phase analysis filters can be allowed.

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