One Structure for Wide-Bandwidth and High-Resolution Fractional Delay Filter

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ABSTRACT

This paper presents one structure for wide-bandwidth and high-resolution FIR fractional delay filters (FDF) with a small number of multipliers per output sample (MPS) and small coefficient computing time. The proposed structure is a modification of the model proposed by J. Vesma at al. Instead of the direct use of the frequency domain design, we use the time domain structure proposed by N. P. Murphy at al. The frequency domain design is made by a least square approximation of each branch filter $C_l(z)$ with an l^{th} order differentiator, but only up to half of the Nyquist frequency. Several design examples confirm that the modified structure has smaller magnitude response ripples and flatter group delay than the original one.

1. INTRODUCTION

The need for a fractional delay filter (FDF) with online adjustable fractional delay value, wide-bandwidth, high fractional delay resolution and small number of multipliers per output sample (MPS) appears in several digital signal processing applications, for example in echo cancellation, phased array antenna systems, timing adjustment in digital receiver, speech coding and synthesis [1].

A polynomial-based FDF allows online fractional delay update by using the Farrow structure [2] or the modified Farrow structure [3]. In figure 1 one such structure, with L+1 branch filters of length N is shown.

In the modified Farrow structure $\gamma=2\alpha-1$, where α denotes the desired fractional delay, $0<\alpha<1$, and the filter coefficients are symmetric (linear phase). On the other hand, in the original Farrow structure $\gamma=\alpha$, and filters do not necessarily have symmetric coefficients.

There are two polynomial-based FDF design approaches. The first one is a time domain design, which uses Lagrange [4] or B-spline interpolators [5]. This design approach can be implemented with the original Farrow structure. One reason of the popularity of this approach is that the polynomial coefficients can be easily determined in a closed form. Its disadvantage is a limited control over the resulting FDF frequency domain response, because only one design parameter, the interpolation polynomial order L, is used.

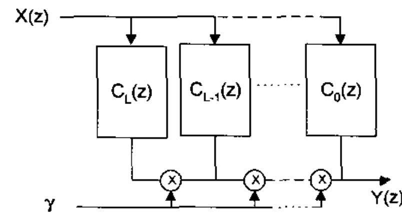

Figure 1 Farrow structure.

The second design approach is a frequency domain design that is made by optimizing each polynomial coefficient directly in the frequency domain. There are several proposed frequency domain design methods, such as [6], [7], where implementation is made with the extended Farrow structure, and in [8] where a Farrow structure is considered. The advantage of the frequency approach is more flexibility in the frequency domain, since three design parameters, polynomial order L, branch filter length N, and passband frequency ω_p can be controlled. The drawback of the frequency domain approach is the necessity of use of an optimization method.

The frequency domain design of a FDF with a wide-bandwidth requires optimization along a wide frequency range. A high fractional delay resolution specification requires a longer branch filter length N and a higher polynomial order L. The above requires an intense optimization computation and a large number of MPS in the resulting FDF.

This paper uses the frequency domain design method proposed by [6], where each branch filter $C_l(z)$ is designed in the frequency domain as a least square approximation of a l^{th} order differentiator.

In order to apply an optimization method more efficiently, we apply the frequency design method to a time domain FDF design structure, proposed in [9].

Next section briefly describes the frequency domain design method. In section 3 the time domain design structure for wide-bandwidth high-resolution FDF is presented. The proposed structure and an illustrative example are presented in section 4.

2. FDF DESIGN BASED ON TAYLOR SERIES

This section briefly reviews the frequency domain FDF design method based on Taylor series. For more details see [6].

The branch filters $C_l(z)$ in Figure 1 have these properties:

- The filters $C_l(z)$ for l=0,1,..L in the Farrow structure form an L^{th} order Taylor series approximation of the continuous-time input signal.
- In the modified Farrow structure, the filters $C_l(z)$ are linear-phase type II when l is even, and type IV when l is odd [3].

Each $C_l(z)$ magnitude response approximately follows the curves $K_l\omega^l$, where the K_l's are constants. Since the ideal frequency response of an l^{th} order differentiator is $(j\omega)^l$, each filter with transfer function $C_l(z)$ in the Farrow structure is an l^{th} order differentiator.

Similarly, the Taylor series approximation can be applied to the modified Farrow structure. This is done by approximating the l^{th} order differential by a linear phase filter $C_l(z)$. Thus, the desired branch filter is given as [6]

$$\hat{C}_l\left(e^{j\omega}\right)=e^{-j\omega\frac{N-1}{2}}\frac{(-j\omega)^l}{2^l l!}. \quad (1)$$

The input parameters of the FDF design method are: the branch filter length N, the polynomial degree L, and the passband frequency ω_p.

The design procedure finds the coefficients of the L+1 transfer functions $C_l(z)$ in such a way that the following error function

$$e_l(\omega)=\left[\sum_{n=0}^{\frac{N}{2}-1}C_l\left(\frac{N}{2}-1-n\right)\gamma(l,n,\omega)-D(l,\omega)\right], \quad (2)$$

is minimized in either the minimax, or the least mean square sense on $[0,\omega_p]$, where,

$$D(l,\omega)=\frac{(-\omega)^l}{2^l l!},$$
$$\gamma(l,n,\omega)=2\cos[(n+1/2)\omega] \quad l \quad even. \quad (3)$$
$$\gamma(l,n,\omega)=2sin[(n+1/2)\omega] \quad l \quad odd$$

In this work the error is minimized in a least mean square sense. Hence, the total error is given as

$$E=\int_0^{\omega_p}\left[\sum_{n=0}^{\frac{N}{2}-1}C_l\left(\frac{N}{2}-1-n\right)\gamma(l,n,\omega)-D(l,\omega)\right]^2 d\omega \cdot \quad (4)$$

We can see from equation (4) that the desired FDF with wide-bandwidth needs an optimization applied in a large frequency range. For a high-resolution specification, a good l^{th} order differentiator is required, which implies a long filter length N and a high polynomial order L.

The number of MPS of the modified Farrow structure is given by

$$MPS1=\left(\frac{N}{2}\right)(L+1)+L. \quad (5)$$

3. WIDE-BANDWIDTH FDF STRUCTURE

This section describes the wide-bandwidth, high-resolution FDF multirate structure, proposed as a time domain design approach in [9].

In figure 2, the basic multirate FDF structure is shown. The upsampled input signal is halfband limited and then passed through a variable FDF designed as a Lagrange interpolator filter [4]. The resulting signal is then decimated by two in order to revert to the original sampling frequency.

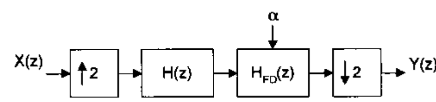

Figure 2 Basic multirate FDF structure.

As it is shown in [9], it is enough to convolve the over-sampled input signal with a fractional delay filter that only meets a desired specification up to half Nyquist frequency.

Since the upsampling operation inserts zeros between the input samples, only half of the interpolation filter coefficients, h(n), are needed. A polyphase structure [10], provides an efficient implementation solution, and is shown in figure 3.

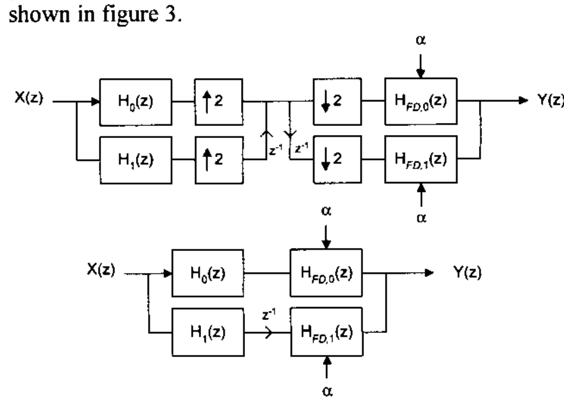

Figure 3 Equivalent multirate FDF structures.

The filters $H_0(z)$ and $H_1(z)$ represent the first and second polyphase components of the filter H(z), respectively. The resulting structure eliminates the need for an upsampler at the input and a downsampler at the output. In order to avoid the one unit delay within the odd coefficient branch, the first and second polyphase components of the FDF, $H_{FD}(z)$, are interchanged [9]. The resulting FDF structure is shown in figure 4.

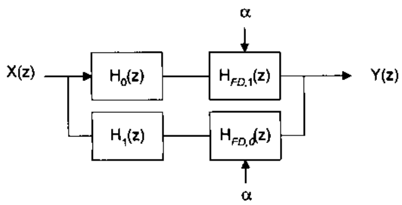

Figure 4 Final single rate FDF structure.

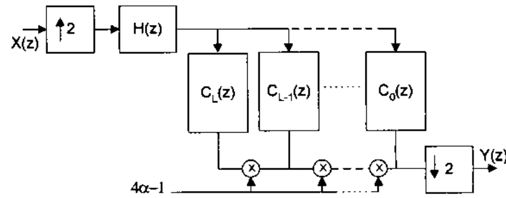

Figure 5 Proposed fractional delay structure.

4. PROPOSED STRUCTURE

The proposed structure combines the frequency optimization FDF design method described in section 2, and the time domain design model discussed in section 3, thereby allowing for a more relaxed FDF specification.

As previously described, the frequency domain optimization is applied from zero up to only half Nyquist frequency. In this way, the computation time is decreased, and the resulting FDF structure has shorter branch filter length N and lower polynomial order L.

The FDF proposed structure is shown in figure 5 where each branch filter $C_l(z)$ is designed as an l^{th} differentiator. The required delay must be scaled by a factor of two, hence $\gamma=4\alpha-1$.

According to Section 3 and the basic multirate identities, the proposed fractional delay structure is computed. It is shown in figure 6, where the filters $C_{l,0}(z)$ and $C_{l,1}(z)$ are the first and second polyphase components of the branch filter $C_l(z)$, respectively.

The interpolation filter H(z) in figure 5 plays a key role in the resulting FDF bandwidth and group delay resolution. The higher the stopband attenuation, the higher the resolution and the smaller transition band, the wider FDF bandwidth. Both conditions imply the need for a high order interpolation filter H(z).

In order to reduce the total number of MPS, this filter is designed as a halfband FIR filter. The second polyphase component $H_1(z)$ is just an integer delay z^{-D}, with integer D given by

$$D=\left\lfloor \frac{N_H+1}{4} \right\rfloor \quad , \tag{6}$$

where N_H is the length of filter H(z). Since the first polyphase component $H_0(z)$ is a symmetric filter, the total number of MPS is significantly decreased. The total complexity in MPS is given by

$$MP2=\left\lfloor (N)(L+1)+\frac{N_H+1}{4}+L \right\rfloor . \tag{7}$$

The proposed structure was implemented in MATLAB. The following example compares the proposed method with the direct use of frequency optimization. The presented coefficient computation time corresponds to the time that the optimization method takes with a number of uniform sampling points equal to 90 in a 800MHz PC.

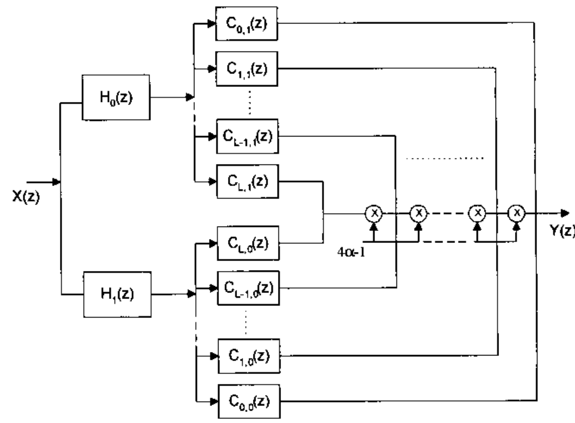

Figure 6 Resulting proposed fractional delay structure

Example: An FDF with a bandwidth of 0.9π and resolution 1/10000th of sample is designed.

The direct use of the frequency domain method, [6], requires L=12 and N= 104 with a total number of MPS of 688 and more than 10 minutes of computation time.

The design parameters used for the discretization-free WLS design method, [8], are: L=7, N=87 and the weighting functions given by

$$W_1(\omega)=\begin{cases}1 & \omega\in[0,0.88\pi), \\ 10 & \omega\in[0.88\pi,0.8994\pi), \\ 0 & \omega\in[0.8994\pi,\pi]\end{cases}$$

$$W_2(p)=1.$$

The resulting number of MPS is 703 and a computing time approximately of 8 seconds.

For the proposed structure, a 241-tap halfband FIR filter, weighted by a 160 dB Dolph-Chebyshev window, was used as the interpolation filter, H(z). For this case the design parameters are L=14 and N=14, with a total number of MPS equal to 254, and a computing time of 21 seconds.

The magnitude responses and group delays for a fractional delay range from 0.0080 to 0.0090 using the direct frequency FDF design method, the discretization free method and the proposed method results are shown in figure 7, figure 8 and figure 9, respectively.

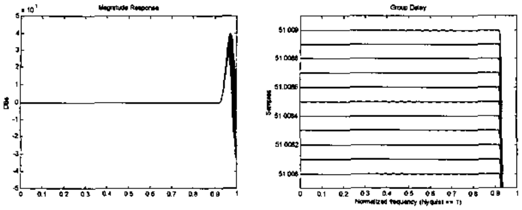

Figure 7 Frequency domain responses for the direct use of the frequency domain design method.

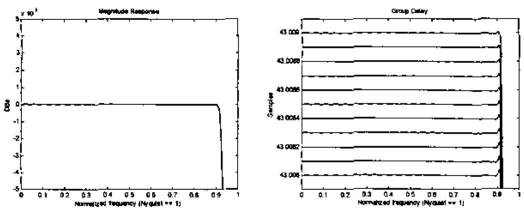

Figure 8 Frequency domain responses for the free-discretization WLS design method.

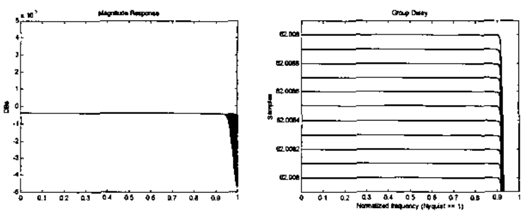

Figure 9 Frequency domain responses for the proposed FDF structure.

We can notice that the proposed FDF structure presents small magnitude responses and very flat group delays. Additionally, there is a saving of 63% in MPS with respect to the direct use of the frequency domain method and a reduction of 64% with respect to the discretization-free method.

5. CONCLUSIONS

One structure for a wide-bandwidth and high-resolution FDF with a small number of MPS and a small coefficient computation time is presented. The improvement is achieved with the use of a multirate structure that allows applying a frequency optimization FDF design method up to only half Nyquist frequency.

In this work, the frequency domain design is made by approximating each branch filter $C_l(z)$ by an l^{th} order differentiator using a least squares optimization, [6]. Analogously, the proposed approach can be applied to any other frequency domain FDF design method.

As the FDF design example presented here shows, besides the notable reduction in the number of MPS and the small computation time, the proposed structure has smaller magnitude response ripples and flatter group delay than the direct use of the FDF frequency domain design method.

The halfband interpolation filter, H(z), in the proposed structure, plays a crucial role in the resulting FDF group delay resolution. The higher resolution and the wider FDF bandwidth are obtained by increasing the stopband attenuation and decreasing the transition band of the filter.

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