ON HIGH-SPEED RECURSIVE DIGITAL FILTERS

Håkan Johansson

Department of Electrical Engineering, Linköping University
SE-581 83 Linköping, Sweden
e-mail: hakanj@isy.liu.se

ABSTRACT

High-speed recursive digital filters are of interest for applications focusing on high-speed as well as low power consumption because excess speed can be traded for low power consumption through the use of power supply voltage scaling techniques. This paper gives an overview of high-speed recursive digital filters. Two different techniques are mainly considered. The first one makes use of interconnected identical allpass subfilters whereas the second one employs frequency masking techniques.

1 INTRODUCTION

The advantage of recursive (IIR) digital filters over their nonrecursive (FIR) counterparts is that they have a lower computational complexity. For frequency selective filters with narrow transition bands the complexity is substantially lower. However, recursive digital filters have a drawback in that they set a bound on the sample frequency at which an implementation of the filters can operate. This may affect not only the speed but also the power consumption since excess speed can be traded for low power consumption by using power supply voltage scaling techniques [1], [2].

1.1 Maximal Sample Frequency

The *maximal sample frequency*, denoted here by f_{max}, of a recursive filter is

$$f_{max} = \min_i \left\{ \frac{N_i}{T_{opi}} \right\} \tag{1}$$

where T_{opi} denotes the total latency of the arithmetic operations, and N_i denotes the number of delay elements, in the directed loop i of the filter structure [3]–[5]. The loop that determines f_{max} is called the critical loop.

1.2 Power Consumption

The major part of the power consumption in a CMOS circuit is due to dynamic power dissipation, which originates from charging and discharging of capacitances. The dynamic power dissipation can be estimated as

$$P = f C_L V_{DD}^2 \tag{2}$$

where f is the switching frequency, C_L is the switched capacitance, and V_{DD} is the power supply voltage [1], [2]. Thus, the power consumption can be substantially reduced by lowering the power supply voltage. However, a reduction of the power supply voltage also increases the propagation delay in the circuits [1], [2]. To be able to reduce the power consumption in an implementation by lowering the supply voltage, it is therefore necessary for the algorithm to have a maximal sample frequency that exceeds the required sample frequency for the application at hand. The technique to trade inherent parallelism in algorithms for low power consumption is referred to as *power supply voltage scaling techniques* [1].

1.3 High-Speed Recursive Digital Filters

We refer to IIR filters with a higher maximal sample frequency than that of conventional filters as *high-speed recursive digital filters*. To obtain a high-speed filter, several techniques can be used. However, the increase in speed is in many cases paid for by an increase of the arithmetic complexity. If the power consumption is to be reduced, the gain that is achieved by using power supply voltage scaling techniques must be larger than the loss that is caused by the increased complexity. It is reasonable to expect that an optimal solution in many cases can be found somewhere between conventional IIR filters and FIR filters. (FIR filters do not have a bound on the maximal sample frequency but, on the other hand, they have a higher arithmetic complexity compared with IIR filters). However, finding an optimal solution is indeed a non-trivial task due to the many aspects that have to be considered when estimating the cost of the final implementation.

Equation (1) reveals two approaches to increase f_{max}. The first is to reduce the latency whereas the second is to increase the number of delay elements in the critical loop. Low latency is obtained by using realizations for which the number of arithmetic operations (i.e., multiplications and additions) is small in the critical loops and for which the latencies of the arithmetic operations are low. The latency of arithmetic operations is highly dependent upon the coefficient word lengths of the filters [2], [6]. Hence, one should use filter structures with low coefficient sensitivity since the coefficient word lengths then will be short. To obtain structures with several delay elements in the critical loops, one can basically use two different approaches, namely algorithm transformation techniques and constrained filter design techniques [7], [8]. Algorithm transformation techniques are based upon pole and zero cancellations [7], [9] which can be achieved theoretically but under finite-arithmetic conditions the cancellations become inexact which may impose problems such as increased coefficient sensitivity and time-variant behavior [10], [11]. Pole and zero cancellations are avoided by using constrained filter design techniques in which the denominator polynomial of the transfer function is restricted to be a function of z^M. The corresponding realization has at least M delay elements in its critical loop, resulting in an M-fold increase of f_{max}.

1.4 Paper Outline

This paper gives an overview of two techniques for obtaining high-speed recursive filters. The first one uses interconnected identical allpass subfilters to obtain low-sensitivity filters and thereby low latency in the critical loops [8], [12], [13]. The second one employs frequency masking (FM) techniques, which belong to the constrained filter design techniques, to obtain filters with several delay elements in the critical loop [7], [8], [14]–[20]. There are several reasons for concentrating on these techniques. For example, the resulting filters have good numerical properties under finite-arithmetic conditions. In particular, it is always possible to obtain robust filters by using wave digital filters (WDFs) [21]–[25] and nonrecursive FIR filters. It is also easy to obtain filters with an approximately linear phase. Further, the resulting filters have in many cases a low arithmetic complexity.

It is difficult to accurately predict the implementation cost of filters and, hence, difficult at the algorithmic level to compare different filters with respect to the power consumption. More work needs to be done in this area. However, by using the techniques discussed in this paper, a variety of different filters with good numerical properties are offered. By considering all of these alternatives, the probability to find at least one filter that satisfies the requirements for the problem at hand increases.

2 LOW-SENSITIVITY FILTERS

Digital filters realized as two allpass subfilters in parallel constitute a favorable class of filters. The transfer function of these filters is

$$H(z) = 0.5[G_0(z) + G_1(z)] \tag{3}$$

where $G_0(z)$ and $G_1(z)$ are stable allpass filters. The corresponding realization is shown in Fig. 1. The allpass filters can, e.g., be realized as a cascade of low-order sections. This results in modular and regular filters which is attractive from the implementation point of view. Further, allpass WDF's can be used which makes it possible to obtain robust filters under finite-arithmetic conditions. The overall filters belong in this case to the well known lattice WDFs [24], [26], [27].

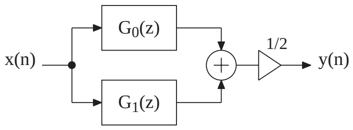

Figure 1. Parallel connection of two allpass filters.

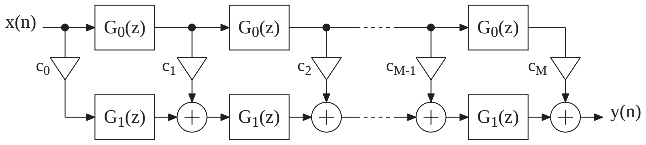

Figure 2. Tapped cascaded interconnection of identical allpass subfilters.

However, filters composed of two allpass filters in parallel have a high coefficient sensitivity in the stopband. This results in long coefficient word lengths and therefore often a lower f_{max} than desired. A simple way to reduce the sensitivity is therefore to use several low-order filters in cascade instead of one single filter [28]. An alternative is to use the more general structure in Fig. 2 [29], [30]. Several other alternatives can be found in [28]. These techniques can also be applied to interpolation and decimation filters [12], filter banks [12], and Hilbert transformers [13].

2.1 Filter Design

The filter in Fig. 2, as well as the interpolation and decimation filters, filter banks, and Hilbert transformers referred to above, can readily be designed using the procedure introduced in [29]. In this approach, the design is split into the design of a nonlinear-phase FIR filter and an IIR filter being realizable as a parallel connection of two allpass filters. For these filters one can use, e.g., the procedure in [33] and the formulas in [27], respectively. For the interpolation and decimation filters, filter banks, and Hilbert transformers, the IIR filter is further restricted to be a half-band IIR filter [12], [13].

2.2 Design Example

Example 1: We consider the structure in Fig. 2 with $M = 6$, $G_0(z) = z^{-1}$, and $G_1(z) = (-\alpha z^2+1)/(z^2-\alpha)$ with $\alpha = -0.75$. The c_i's, $i = 0, 1, ..., M$, are optimized with α fixed using the procedure introduced in [29]. Details are found in [12]. The magnitude response of the overall filter is shown in Fig. 3. The overall filter requires six first-order (in z^2) allpass filters and 7 interconnecting multipliers. However, the multipliers in the allpass filters require only three bits which results in multipliers with low cost and low latency, thus, a high f_{max}. The required number of bits for the c_i's, to keep the stopband attenuation above 60 dB, is 11 using simple rounding. For a corresponding conventional IIR filter, satisfying the same stopband attenuation, the required order is 13. This filter also requires 6 first-order allpass filters but the required number of bits is here much larger (12 bits using simple rounding [12]), resulting in a substantially lower f_{max}.

3 FREQUENCY MASKING FILTERS

Frequency masking techniques employs periodic and nonperiodic model and masking filters [7], [8], [14]–[20]. An advantage of using these techniques is that they offer a large freedom to choose structures for the model and masking filters that are well suited for the specification and problem at hand. For example, WDFs, possibly in combination with nonrecursive FIR filters, can always be used in order to maintain stability under finite-arithmetic conditions. We divide the filters into three different categories.

3.1 Filter Categories

Narrow-Band Filters: Let the transfer function of a narrow-band filter be

$$H(z) = G(z^M)F(z) \tag{4}$$

where $G(z)$ and $F(z)$ here are referred to as *model* and *masking filters*, respectively. The masking filter extracts the desired image, and rejects the undesired images, from the periodic magnitude function (i.e., the magnitude function has a period of $2\pi/M$) of the *periodic model filter* $G(z^M)$. This is illustrated in Fig. 4 for a lowpass filter.

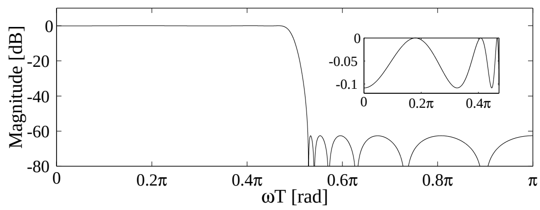

Figure 3. Magnitude response of the overall filter in Example 1.

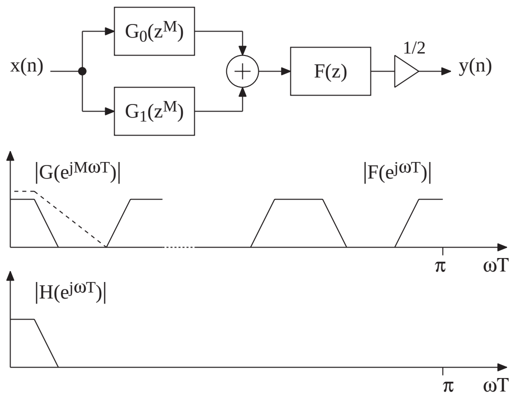

Figure 4. Filter structure and illustration of magnitude functions for high-speed narrow-band filters.

This technique was originally introduced in order to reduce the complexity of FIR filters with narrow transition bands [31]–[32]. (The filters are in this case commonly referred to as interpolated FIR filters.) The simplest way to obtain a high-speed recursive filter is to use an IIR filter for the model filter and an FIR filter for the masking filter [7], [8], [14]. In this case, the denominator polynomial of the transfer function of the IIR filter is a function of z^M which means that the corresponding realization has at least M delay elements in its critical loop, resulting in an M-fold increase of f_{max}. The IIR model filter can, e.g., be realized as a parallel connection of two allpass filters. In this case, the transfer function of $G(z)$ is in the form of (3), i.e., $G(z) = 0.5[G_0(z)+G_1(z)]$. The realization of $H(z)$ is in this case as shown in Fig. 4.

Wide-Band Filters: Let the transfer function of a wide-band filter be

$$H(z) = H_{AP}(z) - G(z^M)F(z) \tag{5}$$

where $H_{AP}(z)$ is an allpass filter as given by

$$H_{AP}(z) = z^{-N/2}G_i(z^M) \tag{6}$$

and i is 0 or 1, $G(z)$ again is $G(z) = 0.5[G_0(z)+G_1(z)]$, and $F(z)$ is an even-order (N) linear-phase FIR filter. The corresponding structure for $i = 0$ is shown in Fig. 5. This type of wide-band filters is thus obtained by parallel connecting a narrow-band filter in the form of (4) with an allpass filter. The transfer function in (5) is the allpass complementary function to that in (4). Typical magnitude responses of the individual and overall filters are as shown in Fig. 5.

Arbitrary Bandwidth Filters: Let the transfer function of an arbitrary bandwidth filter be in the form of

$$H(z) = G(z^M)F_0(z) + G_c(z^M)F_1(z) \tag{7}$$

where $G(z)$ again is $G(z) = 0.5[G_0(z)+G_1(z)]$, $G_c(z)$ is its complementary filter given by $G_c(z) = 0.5[G_0(z)-G_1(z)]$, and $F_0(z)$ and $F_1(z)$ are linear-phase FIR filters of equal delays. The corresponding structure for $i = 0$ is shown in Fig. 6. For arbitrary bandwidths, we thus need two masking filters which extract one or several passbands of the periodic model filters $G(z^M)$ and $G_c(z^M)$, respectively, as illustrated in Fig. 6 for a lowpass filter.

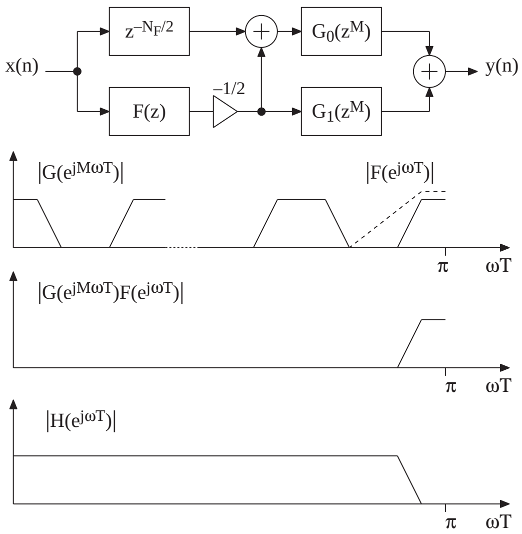

Figure 5. Filter structure and illustration of magnitude functions for high-speed wide-band filters.

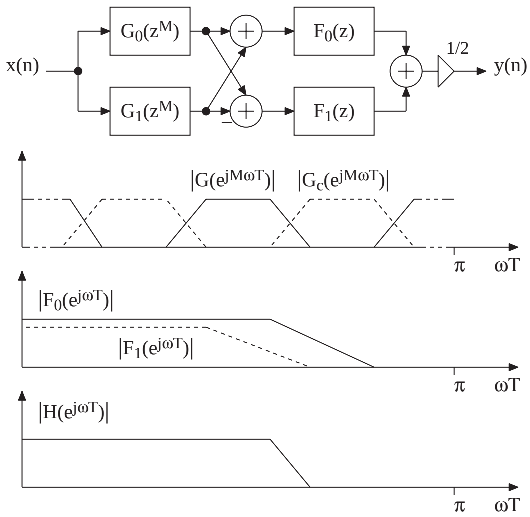

Figure 6. Filter structure and illustration of magnitude responses for high-speed arbitrary bandwidth filters.

3.2 Filter Design

Satisfying magnitude response requirements: For all three filter categories, the overall filter can easily be designed to meet some prescribed magnitude response requirements by separately optimizing an IIR model filter, being realizable as a parallel connection of two allpass filters, and one or two linear-phase FIR masking filters using well known approximation techniques. If the phase response is of less importance, one should use a Cauer filter for the IIR model filter in order to minimize the overall complexity.

Satisfying phase response requirements: For all three filter categories, the overall filter can be designed to meet some prescribed requirements on the phase response as well, by restricting one of the allpass branches of the IIR model filter to be a pure delay [16]–[20]. The IIR model filter can in the this case be designed using, e.g., the algorithm in [34].

Further Optimization: By optimizing the model and masking filters separately, a simple and fast design procedure is offered, but the overall filter is not optimal. It can therefore be beneficial to consider simultaneous optimization of the model and masking filters in order to improve the result or reduce the computational complexity [18]–[20]. One way of doing this is to use the filters obtained in the approach of separate optimization as initial filters in some standard nonlinear optimization routine.

3.3 Computational Complexity

In the nonlinear-phase case, the increased $f_{\max}$ is paid for by an increased arithmetic complexity. However, in many cases the increased complexity is fairly modest, especially for narrow-band and wide-band filters. Further, compared with high-speed filters based of algorithm transformation techniques, FM filters are indeed competitive and often advantageous [8], [20].

In the approximately linear-phase case, the overall complexity will generally reach a minimum for a certain value of M that need not to be $M = 1$. This resembles the case in which only FIR filters are used [32], [35]. For these filters, we can thus obtain both an increased $f_{\max}$ as well as a reduced complexity. This is demonstrated in Example 2 in Section 3.6.

3.4 Extensions

Above, we have only considered FM filters with IIR model and FIR masking filters. For narrow-band and wide-band filters, the FM techniques have been extended to the case where both the model and masking filters are IIR filters [8], [16], [18], mainly in order to reduce the overall complexity. Recently, FM combined with multirate techniques were considered in [36] in order to further reduce the complexity. The FM techniques have also been extended to interpolation and decimation filters [19], and Hilbert transformers [8].

Table 1. Results of Example 2.

	Order	Mult	$\tau_{g\,av}$	$\Delta\tau_g$	$\Phi_{e\,\max}$	$f_{\max\,norm}$
M = 1 (Conventional) H(z) = G(z)	51	26	25	2.02	0.010	1
M = 2						
G(z)	27	14	13			
F(z)	15	8	7.5			
H(z)	69	22	33.5	2.04	0.010	2
M = 4						
G(z)	15	8	7			
$F_0(z)$	17	9	8.5			
$F_1(z)$	17	9	8.5			
H(z)	77	26	36.5	1.85	0.0087	4
M = 6						
G(z)	11	6	5			
$F_0(z)$	17	9	8.5			
$F_1(z)$	49	25	24.5			
H(z)	115	40	54.5	0.85	0.0037	6

3.5 Related Techniques

It should be pointed out that there exist other constrained filter design techniques than FM techniques. A number of such techniques, in which the denominator polynomial of the transfer function is restricted to be a function of z^M, have over the past decades been proposed mainly in order to obtain efficient interpolators, decimators, and multirate narrow-band filters, see e.g. [37]–[40]. However, in addition to being suitable only for certain narrow-band filters these techniques have a drawback in that they restrict the choice of filter structures because the transfer function is expressed as $H(z) = N(z)/D(z^M)$.

3.6 Design Example

This example illustrates that we can achieve a reduced complexity as well as an increased maximal sample frequency for the approximately linear-phase filters. One price to pay is however that the overall delay is increased.

Example 2: Consider a lowpass filter meeting the following specification: $\omega_c T = 0.35\pi$ rad, $\omega_s T = 0.4\pi$ rad, $A_{max} = 0.2$ dB, $A_{min} = 40$ dB, and $\Phi_e(\omega T)$ 0.01 rad in the passband (where $\Phi_e(\omega T)$ is the phase error [20]). We study the conventional filter ($M = 1$) the structure in Fig. 4 with $M = 2$ and the structure in Fig. 6 with $M = 4$, and $M = 6$. The model and masking filters are designed separately using equiripple approximations. Details can be found in [20]. The results of the different designs are compiled in Table 1. The magnitude responses for $M = 1$, $M = 4$, and $M = 6$ are shown in Fig. 7. The phase errors for the same filters are plotted in Fig. 8.

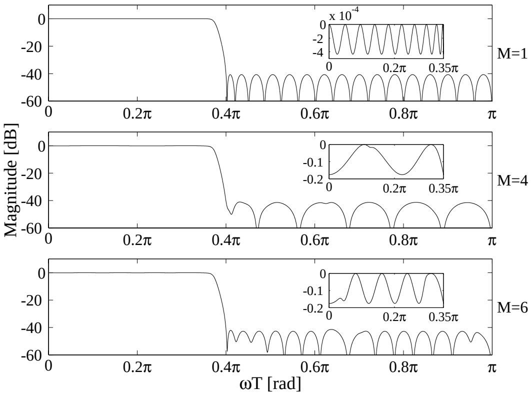

Figure 7. Magnitude responses for the overall filters in Example 2.

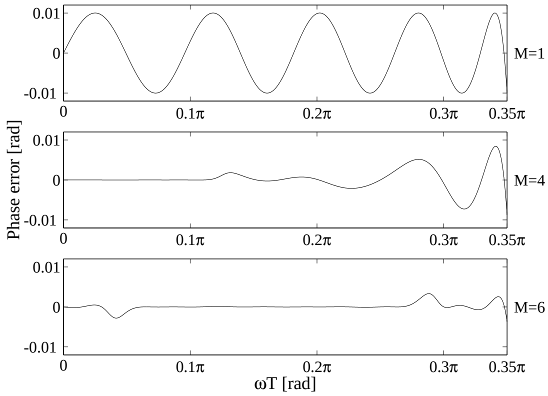

Figure 8. Phase errors for the overall filters in Example 2.

REFERENCES

[1] A. P. Chandrakasan and R. W. Brodersen: *Low Power Digital CMOS Design*, Norwell, MA: Kluwer, 1995.

[2] M. Vesterbacka: *On Implementation of Maximally Fast Wave Digital Filters*, Linköping Studies in Science and Technology, Diss. no. 487, Linköping University, Sweden, June 1997.

[3] R. Reiter, "Scheduling parallel computations," *J. Ass. Comp. Machn.*, vol. 15, no. 4, pp. 590-599, Oct. 1968.

[4] A. Fettweis, "Realizability of digital filter networks," *Arch, Elektr. Übertragung.*, vol. 30, no. 2, pp. 90–96, Feb. 1976.

[5] M. Renfors and Y. Neuvo, "The maximum sampling rate of digital filters under hardware speed constraints," *IEEE Trans. Circuits Syst.*, vol. CAS-28, no. 3, pp. 196-202, March 1981.

[6] L. Wanhammar, *DSP Integrated Circuits*, New York: Academic, 1999.

[7] J. G. Chung and K. K. Parhi, *Pipelined Lattice and Wave Digital Recursive Filters*, Norwell, MA: Kluwer, 1996.

[8] H. Johansson: *Synthesis and Realization of High-Speed Recursive Digital Filters*, Linköping Studies in Science and Technology, Diss. no. 534, Linköping University, Sweden, May 1998.

[9] K. K. Parhi and D. G. Messerschmitt, "Pipeline interleaving and parallelism in recursive digital filters—Part I: Pipelining using scattered look-ahead and decomposition," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-37, no. 7, pp. 1099–1117, July 1989.

[10] K. S. Arun, and D. R. Wagner, "High-speed digital filtering: Structures and finite wordlength effects," *J. VLSI Signal Processing*, vol. 4, no. 4, pp. 355-370, Nov. 1992.

[11] Y. Jang and S. P. Kim, "Block digital filter structures and their finite precision responses," *IEEE Trans. Circuits Syst. II*, vol. 43, no. 7, pp. 495-506, July 1996.

[12] H. Johansson and L. Wanhammar, "High-speed recursive filter structures composed of identical allpass subfilters for interpolation, decimation, and QMF banks with perfect magnitude reconstruction," *IEEE Trans. Circuits Syst. II*, vol. 46, no. 1, pp. 16-28, Jan. 1999.

[13] H. Johansson and L. Wanhammar, "Digital Hilbert transformers composed of identical allpass subfilters," in *Proc. IEEE Int. Symp. Circuits Syst.*, Monterey, CA, May 31-June 3, 1998, vol. V, pp. 437-440.

[14] J. G. Chung and K. K. Parhi, "Pipelined wave digital filter design for narrow-band sharp-transition digital filters," in *Proc. IEEE Workshop VLSI Signal Processing*, La Jolla, CA, Oct. 1994, pp. 501-510.

[15] J. G. Chung, H. Kim, and K. K. Parhi, "Pipelined lattice WDF design for wideband filters," *IEEE Trans. Circuits Syst. II*, vol. 42, no. 9, pp. 616-618, Sept. 1995.

[16] H. Johansson and L. Wanhammar, "Filter structures composed of allpass subfilters for high-speed narrow-band and wideband filtering," in *Proc. European Conf. Circuit Theory and Design*, Budapest, Hungary, Aug. 31-Sept. 3, 1997, vol. 2, pp. 561-566.

[17] H. Johansson and L. Wanhammar, "A digital filter structure composed of allpass filters and an FIR filter for wideband filtering," in *Proc. IEEE Int. Conf. Electronics, Circuits, Syst.*, Cairo, Egypt, Dec. 15-18, 1997, vol. 1, pp. 249-253.

[18] H. Johansson and L. Wanhammar, "Wave digital filter structures for high-speed narrow-band and wideband filtering," *IEEE Trans. Circuits Syst. II*, vol. 46, no. 6, pp. 726-741, June 1999.

[19] H. Johansson and L. Wanhammar, "Filter structures composed of allpass and FIR filters for interpolation and decimation by a factor of two," *IEEE Trans. Circuits Syst. II*, vol. 46, no. 7, pp. 896-905, July 1999.

[20] H. Johansson and L. Wanhammar, "High-speed recursive digital filters based on the frequency-response masking approach," *IEEE Trans. Circuits Syst. II*, vol. 47, no. 1, pp. 48-61, Jan. 2000.

[21] A. Fettweis and K. Meerkötter, "Suppression of parasitic oscillations in wave digital filters," *IEEE Trans. Circuits Syst.*, vol. CAS-22, no. 3, pp. 239-246, Mar. 1975.

[22] T. A. C. M. Claasen, W. F. G. Mecklenbräuker, and J. B. H. Peek, "On the stability of the forced response of digital filters with overflow nonlinearities," *IEEE Trans. Circuits Syst.*, vol. CAS-22, no. 8, pp. 692-696, Aug. 1975.

[23] K. Meerkötter, "Incremental pseudopassivity of wave digital filters," in *First European Signal Processing Conf.*, pp. 27-31, Lausanne, Switzerland, Sept. 1980.

[24] A. Fettweis, "Wave digital filters: Theory and practice," *Proc. IEEE*, vol. 74, no. 2, pp. 270-327, Feb. 1986.

[25] A. Fettweis, "On assessing robustness of recursive digital filters," *Eur. Trans. Telecomm. Relat. Technol.*, (Italy), vol. 1, no. 2, pp. 103-109, Mar.-Apr., 1990.

[26] A. Fettweis, H. Levin, and A. Sedlmeyer, "Wave digital lattice filters," *Intern. J. Circuit Theory and Appl.*, vol. 2, pp. 203-211, June 1974.

[27] L. Gazsi, "Explicit formulas for lattice wave digital filters," *IEEE Trans. Circuits Systems*, vol. CAS-32, no. 1, pp. 68-88, Jan. 1985.

[28] J. Yli-Kaakinen and T. Saramäki, "Design of very low-sensitivity and low-noise recursive filters using a cascade of low-order lattice wave digital filters," *IEEE Trans. Circuits Syst. II*, vol. 46, no. 7, pp. 906-914, July 1999.

[29] T. Saramäki and M. Renfors, "A novel approach for the design of IIR filters as a tapped cascaded interconnection of identical allpass subfilters," in *Proc. IEEE Int. Symp. Circuits Syst.*, vol. 2, pp. 629-632, Philadelphia, May 4-7, 1987.

[30] H. Johansson and T. Saramäki, "A class of complementary IIR filters," in *Proc. IEEE Int. Symp. Circuits Syst.*, Orlando, Florida, May 30-June 2, 1999, vol. 3, pp. 299-302.

[31] Y. Neuvo, D. Cheng-Yu, and S. K. Mitra, "Interpolated finite impulse response filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-32, no. 3, pp. 563-570, June 1984.

[32] T. Saramäki, Y. Neuvo, and S. K. Mitra, "Design of computationally efficient interpolated FIR filters," *IEEE Trans. Circuits Syst.*, vol. 35, no. 1, pp. 70-88, Jan. 1988.

[33] O. Herrman and H. W. Schussler, Design of Nonrecursive Digital Filters with Minimum Phase, Electronics Letters, Vol. 6, No. 11, pp. 329-330, May 1970.

[34] M. Renfors and T. Saramäki, "A class of approximately linear phase digital filters composed of allpass subfilters," in *Proc. IEEE Int. Symp. Circuits Syst.*, San Jose, CA, May 1986, pp. 678-681.

[35] Y. C. Lim, "Frequency-response masking approach for the synthesis of sharp linear phase digital filters," *IEEE Trans. Circuits Syst.*, vol. CAS-33, no. 4, pp. 357-364, April 1986.

[36] H. Johansson, "Multirate single-stage and multistage structures for high-speed recursive digital filtering," in *Proc. IEEE Int. Symp. Circuits Syst.*, Orlando, Florida, May 30-June 2, 1999, vol. 3, pp. 291-294.

[37] H. G. Martinez and T. W. Parks, "A class of infinite-duration impulse response digital filters for sampling rate reduction," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-27, no.2, pp. 154-162, Apr. 1979.

[38] M. A. Richards, "Application of Deczky's program for recursive filter design to the design of recursive decimators," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-30, pp. 811-814, Oct. 1982.

[39] S. Chu and C. S. Burrus, "Optimum FIR and IIR multistage multirate filter design," *Circuits, Syst., Signal Processing*, vol. 2, no. 3, pp. 361-385, 1983.

[40] T. Saramäki, "Design of optimal multistage IIR and FIR filters for sampling rate alteration," in *Proc. IEEE Int. Symp. Circuits Syst.*, San Jose, CA, May 5-7, 1986, vol. 1, pp. 227-230.