A Simplified Structure for FIR Filters with an Adjustable Fractional Delay

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Adjustable Fractional Delay (AFD) Filters

Why AFD filters?

- In various applications, there is a need for a delay that is a fraction of a sampling interval.
- Furthermore, it is often desired that the delay value is adjustable.

Desired frequency response:

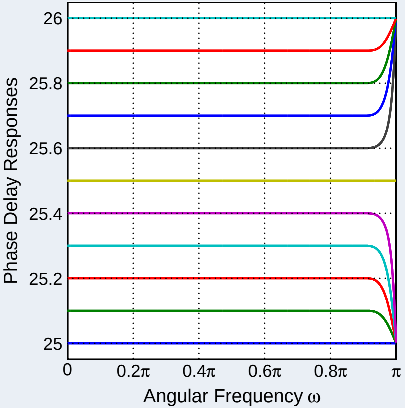
$$H_{\text{des}}(e^{j\omega},\ \mu) = e^{-j\omega(D-1+\mu)},$$

where D is an integer delay and μ is an adjustable fractional delay in the range [0, 1].

Example phase delay responses

$$\tau(\omega, \mu) = -\arg H(e^{j\omega}, \mu)/\omega$$

for $\mu = 0, 0.1, 0.2, \ldots, 1$.



Classes of Adjustable Fractional Delay Filters

These filters can be designed either using FIR or IIR filters

Farrow structure consisting of several parallel fixed FIR filters:

Outputs of these filters are multiplied with quantities depending on the value of the fractional delay.

All-pass gathering structure proposed by Makundi *et al.*:

The filter coefficients are the polynomials of the desired value of the fractional delay.

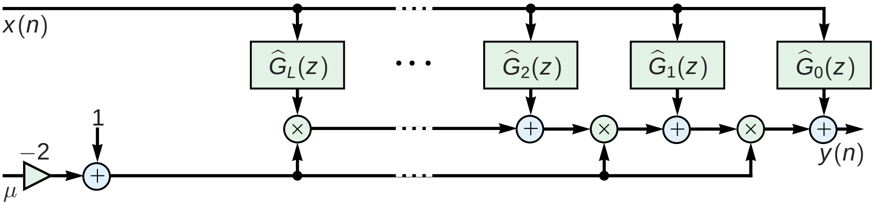


Figure: Modified Farrow structure for AFD FIR filters.

Proposed Structure

Overall filter structure

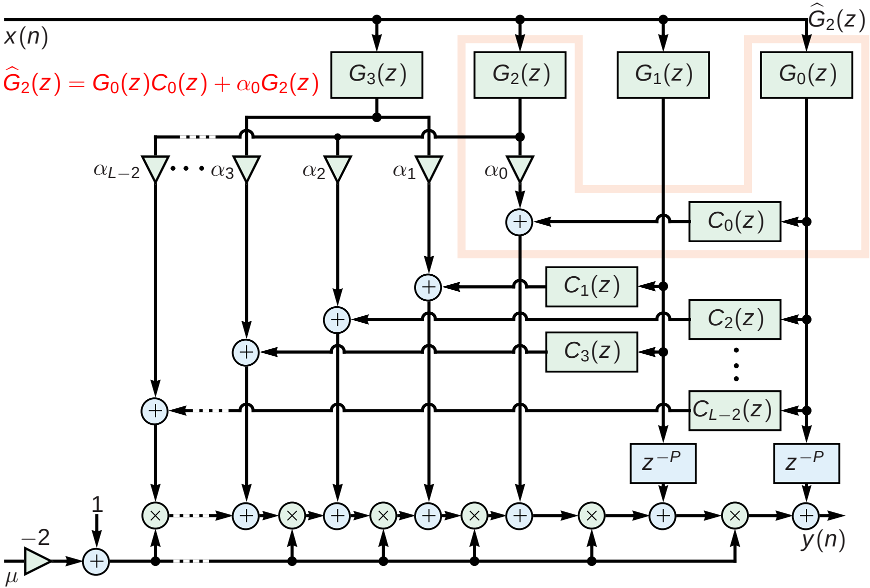
In the proposed structure the first two subfilters are the same as in the modified Farrow structure, whereas the remaining ones are generated by properly combining these two subfilters with some additional very short filters, adders, multipliers, and pure delay terms.

Relation to the original modified Farrow structure

The relations between the subfilters of the original modified Farrow structure and the prosed one are given as

$$\widehat{G}_l(z) = \begin{cases} G_0(z)z^{-P} & \text{for } l = 0 \\ G_1(z)z^{-P} & \text{for } l = 1 \\ G_0(z)C_{l-2}(z) + \alpha_{l-2}G_2(z) & \text{for } l \text{ even and } l \neq 0 \\ G_1(z)C_{l-2}(z) + \alpha_{l-2}G_3(z) & \text{for } l \text{ odd and } l \neq 1. \end{cases}$$

Proposed Structure



Proposed Structure

The transfer functions of the subfilters

The transfer functions of the two first FIR filters are of the form

$$G_{0,1}(z) = \sum_{n=0}^{M-1} g_0(n) z^{-n} \pm \sum_{n=M}^{2M-1} g_0(2M-1-n) z^{-n},$$

respectively.

The transfer functions with the plus and with the minus sign are the transfer functions of $G_0(z)$ and $G_1(z)$, respectively.

The transfer functions of the remaining two FIR filters are given in a same manner by

$$G_{2,3}(z) = z^{-(M+P-1)} \pm z^{-(M-P)},$$

respectively.

Optimization Problem

Optimization goal

Determine the adjustable parameters such that for each value of μ within $0 \leq \mu \leq 1$ the phase delay closely approximates $M + P - 1 + \mu$.

Statement of the problem

Given the filter specifications

1) the passband edge ω_p, and
2) the phase delay and magnitude ripples δ_p and δ_a, respectively.

Find

a) the subfilter orders $2M$, b) the number and the orders of additional filters $L - 1$ and $2P$, respectively, and c) the filter coefficient values

so that

1) the maximum absolute values of the phase delay and magnitude errors are minimized and
2) the implementation complexity is minimized.

Three-Step Optimization Algorithm

Coefficient optimization is performed in three stages:

Step 1: Find the minimum value of M so that the magnitude response of $G_0(z)$ approximates unity for $\omega \in \Omega_p$ such that the deviation from unity is less than $\zeta\delta_a$. (ζ is within 0.5 and 0.7).

Step 2: Design $G_1(z)$ and increase $L+1$ the overall number of branch filters, and design $C_l(z)$ for $l = 0, 1, \ldots, L-2$ as well as additional coefficients until the criteria are met by $\gamma\delta_a$ and $\gamma\delta_p$. (γ is around 0.75.)

Step 3: Exploit the following observation and re-optimize the remaining parameters to meet the given criteria.

Experimentally observed fact:

Some coefficients values have negligible effect on the filter performance and can be fixed to be zero valued.

Three-Step Optimization Algorithm

Step 2 can be accomplished as follows:

1st: $G_1(z)$ of the same order $2M-1$ is designed such that the frequency response of $G_1(z)$ approximates for $\omega \in \Omega_p$ the response of first-order differentiator. (Remez)

2nd: $C_l(z)$'s and the additional coefficients α_l's are optimized such that the following maximum absolute error between the resulting and the desired frequency responses is minimized: (SDP)

$$\xi_l = \max_{\omega \in \Omega_p} \left| G_n(e^{j\omega}) C_l(e^{j\omega}) + \alpha_l G_m(e^{j\omega}) - D_{l+2}(e^{j\omega}) \right|.$$

Here $D_l(e^{j\omega})$ is the frequency response of the lth-order differentiator and $G_n(e^{j\omega})$ and $G_m(e^{j\omega})$ are $G_0(e^{j\omega})$ and $G_2(e^{j\omega})$ for l even, respectively, and $G_1(e^{j\omega})$ and $G_3(e^{j\omega})$ for l odd, respectively.

The minimum orders for the $C_l(z)$'s are determined by gradually increasing the order until certain criteria are met.

Example 1: $\Omega_p = [0, 0.9\pi]$, $\delta_a \leq 0.01$, and $\delta_p \leq 0.001$

Step 1: Estimating subfilter orders

The minimum odd order for which the magnitude response of $G_0(z)$ stays within $1 \pm \zeta\delta_a$ on Ω_p becomes 27 $(M = 14)$.

The worst-case magnitude error for this design is 0.005 02.

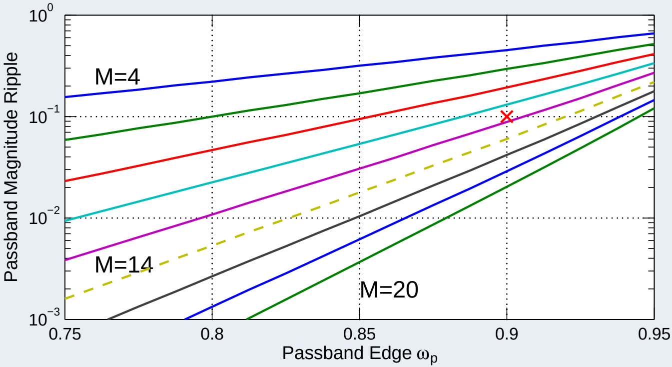


Figure: Magnitude errors as functions of passband edge for $M = 4, 6, \ldots, 20$.

Example 1: $\Omega_p = [0, 0.9\pi]$, $\delta_a \leq 0.01$, and $\delta_p \leq 0.001$

Step 2: Finding the initial coefficients values

For the $P = 0$ and $P = 1$ designs $\xi_0 = 0.028\,043$ and $\xi_0 = 0.008\,148$, respectively, that is the minimum order for the $C_l(z)$'s becomes two.

For the $L = 3$ design, Δ_a, the worst-case magnitude error, and Δ_p, the worst-case phase delay error, became $\Delta_a = 10\Delta_p = 0.030\,443$. This indicates that L should be increased.

The minimum value of L required to meet the given overall criteria became four, that is, the overall structure consist three filters $C_l(z)$ for $l = 0, 1, 2$ of order $2P = 2$.

For the optimized overall design, $\Delta_a = 10\Delta_p = 0.006\,825$.

Example 1: $\Omega_p = [0, 0.9\pi]$, $\delta_a \le 0.01$, and $\delta_p \le 0.001$

Step 3: Reducing the implementation complexity

Carried out by gradually fixing coefficients to zero and reoptimizing.

k	$g_0(k)$	$g_1(k)$	$g_0(k)$	$g_1(k)$
0	−0.001 13	0.002 76	−0.002 63	0
1	0.006 15	−0.003 58	0.003 08	0
2	−0.005 77	0.002 54	−0.006 07	0
3	0.009 23	−0.003 69	0.008 90	0
4	−0.012 91	0.004 25	−0.012 36	0
5	0.017 80	−0.004 82	0.016 67	0
6	−0.024 06	0.005 21	−0.022 23	0
7	0.032 20	−0.005 27	0.029 78	0
8	−0.043 05	0.004 73	−0.043 38	0
9	0.058 15	−0.003 00	0.059 98	0.003 94
10	−0.080 86	−0.001 35	−0.079 67	−0.010 21
11	0.119 96	0.013 16	0.120 69	0.021 80
12	−0.207 72	−0.057 89	−0.207 42	−0.066 72
13	0.635 11	0.623 91	0.635 75	0.632 53

$c_0(0) = -0.006\,23$
$c_0(1) = -1.167\,71$
$c_0(2) = -0.006\,23$

$c_1(0) = 0$
$c_1(1) = -0.997\,16$
$c_1(2) = 0$

$c_2(0) = 0$
$c_2(1) = 0.148\,59$
$c_2(2) = 0$

$\alpha_0 = 0.597\,86$
$\alpha_1 = 0.494\,75$
$\alpha_2 = -0.086\,70$

Can be fixed to zero valued.

$\delta_a = 10\delta_p = 0.009\,594$

Comparison with other AFD FIR Filters

Example 1: $\Omega_p = [0,\ 0.9\pi]$, $\delta_a \leq 0.01$, and $\delta_p \leq 0.001$

Structure	$\delta_a = 10\delta_p$	M	L	P	N_{M}
Modified Farrow [4]	0.006 571	13	4	–	69
Modified Farrow [10]	0.005 608	14	5	–	57
Modified Farrow [5]	0.009 069	14	4	–	32
Proposed	0.009 501	14	4	1	30

N_{M} denotes the number multipliers.

[4] Vesma and Saramäki, "Optimization and efficient implementation of FIR filters with adjustable fractional delay," in *Proc. ISCAS*, Hong Kong, 1997.

[10] H. Johansson and P. Löwenborg, "On the design of adjustable fractional delay FIR filters," *IEEE Trans. Circuits Syst. II*, vol. 50, pp. 164–169, 2003.

[5] J. Yli-Kaakinen and T. Saramäki, "Multiplier-free polynomial-based FIR filters with an adjustable fractional delay," *Circuits, Syst., Signal Process.*, vol. 25, pp. 265–294, 2006.

Comparison with other AFD FIR Filters

Example 2: $\Omega_p = [0,\ 0.9\pi]$, $\delta_a = \delta_p \leq 10^{-4}$

Structure	$\delta_a = \delta_p$	M	L	P	N_M
Modified Farrow [10]	$\approx 10^{-5}$	38	10	–	250
Modified Farrow [11]	$\approx 10^{-5}$	25	13	–	227
Proposed	$7.82 \cdot 10^{-6}$	34	9	3	100

N_M denotes the number multipliers.

[10] H. Johansson and P. Löwenborg, "On the design of adjustable fractional delay FIR filters," *IEEE Trans. Circuits Syst. II*, vol. 50, pp. 164–169, 2003.

[11] E. Hermanowich, "On designing a wideband fractional delay filter using the Farrow approach," in *Proc. Euro. Signal Process. Conf.*, Vienna, Austria, Sept. 2004, pp. 961–964.

Phase Delay and Magnitude Responses in Ex. 2

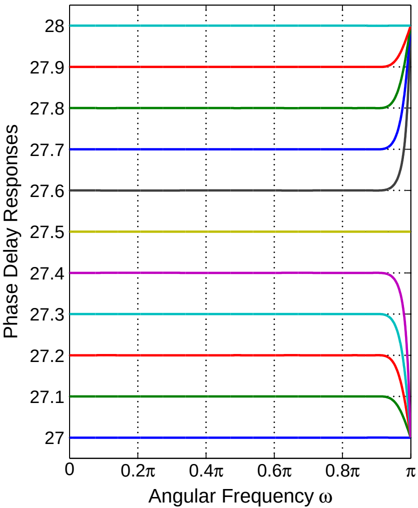


Figure: Phase Delay Responses

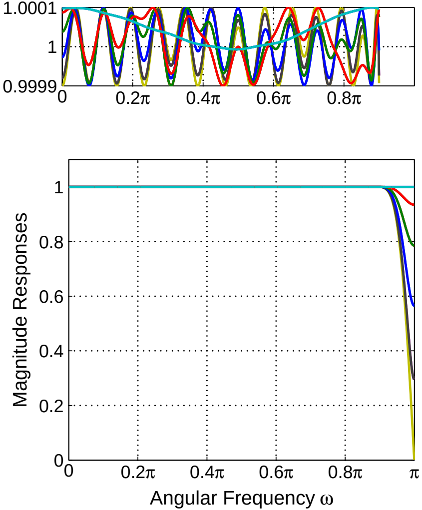


Figure: Magnitude Responses

Further Reading

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- J. Vesma and T. Saramäki, "Optimization and efficient implementation of FIR filters with adjustable fractional delay," in *Proc. IEEE Int. Symp. Circuits Syst.*, vol. 1, Hong Kong, June 9–12 1997, pp. 2256–2259.
- H. Johansson and P. Löwenborg, "On the design of adjustable fractional delay FIR filters," *IEEE Trans. Circuits Syst. II*, vol. 50, pp. 164–169, Apr. 2003.