

# Timing Offset Compensation in Fast-Convolution Filter Bank Based Waveform Processing

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**Abstract**—Fast convolution processing has recently been proposed as an efficient means for implementing filter bank multicarrier systems with good spectral containment and high flexibility in tuning the subchannel bandwidths and center frequencies. These features make fast convolution filter banks (FC-FBs) a particularly interesting choice for multicarrier transmission in challenging radio scenarios like dynamic spectrum access, cognitive radio, and fragmented spectrum use. The FC-FB approach supports also non-synchronized operation of different users, with high spectral efficiency, which is also a highly desirable feature in the considered scenarios. In this paper we explore the possibilities of compensating timing offsets in FC-FB processing by adjusting the FFT-domain weights. It is demonstrated that arbitrary fractional timing offsets can be handled with low complexity and without significant loss in system performance.

## I. INTRODUCTION

Multicarrier modulation has most of the key elements needed in the challenging new spectrum use scenarios, like opportunistic dynamic spectrum access, cognitive radio, and heterogeneous wireless system coexistence. Characteristic to these situations is the need to adjust the spectral characteristics of the transmitted signal, notably bandwidth and center frequency, to the available unused slots of radio spectrum. To support high data rates, it is often desirable to combine multiple non-contiguous spectrum slots in the transmission. In multicarrier systems, this can be achieved by activating only those subcarriers that are within the available frequency slots. One important example case of such fragment spectrum use is the high data rate services to be developed for the Professional Mobile Radio (PMR), which is utilized, e.g., by safety organizations [1]. Currently, PMR supports voice and narrowband data services, based on the family of Tetra standards in Europe and many countries outside Europe, as well as APCO25 in North America.

Orthogonal frequency-division multiplexing (OFDM) is the most important multicarrier technique and it is extensively utilized in modern broadband radio access systems. This is due to the simple and robust way of doing channel equalization, high flexibility and efficiency in allocating spectral resources to different users, as well simplicity of combining multiantenna schemes with the core functionality [2]. However, OFDM has one major limitation in the mentioned co-existence scenarios: limitations in spectral containment, which leads to high sensitivity to interferences from asynchronous spectral components, e.g., in fragmented spectrum use.

An alternative scheme for the considered scenarios is offered by the filter bank based methods of waveform processing

and channelization filtering [3]. Actually, it is possible to combine both functions in filter bank based implementations:

- 1) The waveforms generated for transmission are spectrally well-contained and no other measures are needed to clean the unused portions of the spectrum allocated for dynamic/fragmented use.
- 2) The filter bank processing on the receiver side is able to suppress the interferences in the unused parts of the allocated spectrum.

Naturally, there are limitations in the reachable levels of attenuation, mostly determined by the analog RF imperfections, notably power amplifier nonlinearity on the transmitter side and nonlinearity of the active stages of the receiver chain, as well as I/Q imbalance effects on both sides [4].

A relatively widely studied filter bank based waveform is FBMC/OQAM (filter bank multicarrier/offset-QAM, more commonly known as OFDM /OQAM) [5] [6]. While reaching high spectral containment, it keeps many of the important features of OFDM. Even though FBMC/OQAM has its limitations in terms of conceptual and implementation complexity and difficulties with certain multiantenna transmission schemes, it has received increasing interest in the mentioned challenging spectrum use scenarios. There are also filter bank based single carrier transmission schemes with high flexibility and reduced peak-to-average power ratio (PAPR), similar to the OFDM based SC-FDMA adopted for the LTE uplink [7].

Apart from spectral isolation of different services in heterogeneous networks, another important issue is the required level of synchronization between different users of the broadband system. This is important since the system is expected to support also direct mobile-to-mobile or ad-hoc networking scenarios. In OFDM based multiple access, OFDMA, the relative frequency offsets should be in the order of one percent of the subcarrier spacing, or smaller, and timing offsets, combined with channel delay spreads, should be within the cyclic prefix. To reach such a level of synchronism, relatively complicated synchronization procedures are required and all stations need to be under tight control of a base station, also in the idle mode. On the other hand, in FBMC/OQAM, different groups of subchannels can be well isolated from each other, by using a narrow guardband of just one subcarrier, and fully asynchronous operation is possible with high spectral efficiency. However, traditional FBMC/OQAM schemes have limited capability to support reception of asynchronous multiplexes of subcarriers. Specifically, timing offsets can be compensated by using multitap subcarrier equalizers. But for realistic equalizer lengths, the timing offset should be small

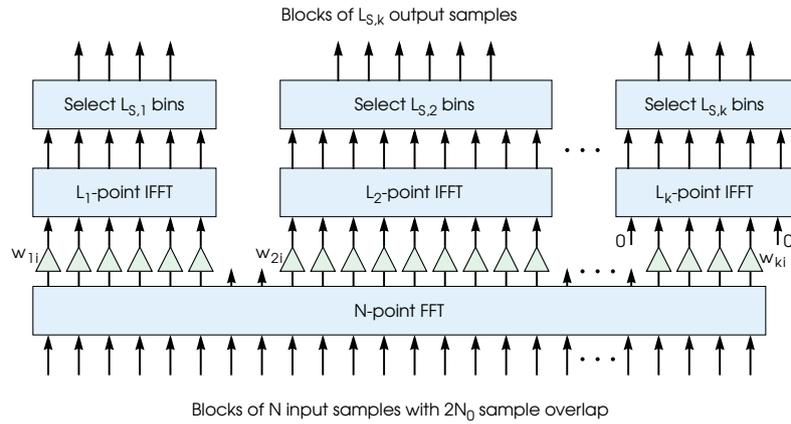


Fig. 1. Fast convolution based flexible analysis filter bank using overlap-save processing.

with respect to the symbol interval [8]. On the other hand, since an FC based analysis FB consists of an FFT, frequency-domain weights, and an IFFT, timing offset compensation can be included by adjusting the phases of the weights. Basically, a linear phase response corresponding to the needed time shift needs to be combined with the weight values resulting from the basic FC-FB design. In this paper we explore how well this idea performs in practice.

After introducing the ideas of FC-FB in Section II, the idea of timing offset compensation using the FC-FB structure is presented in Section III. A numerical case study about the resulting performance is presented in Section IV. This study is based on the parameters of the 5 MHz LTE system, which is commonly considered as a reference basis in the PMR/PPDR (Public Protection and Disaster Relief) system development due to its modularity and wide adoption in the civil world.

## II. FAST-CONVOLUTION FILTER BANKS

This paper focuses on a special implementation scheme for multirate filter banks which is based on fast-convolution (FC) processing. The basic idea of fast convolution is that a high-order filter can be implemented effectively through multiplication in frequency domain, after taking DFT's of the input sequence and the filter impulse response. Eventually, the time domain output is obtained by IDFT. Commonly, efficient implementation techniques, like FFT/IFFT, are used for the transforms, and overlap-save processing is adopted for processing long sequences. The application of FC to multirate filters has been presented in [9], and FC implementations of channelization filters have been considered in [10], [11], [12]. The authors have introduced in [13] the idea of FC-implementation of nearly perfect-reconstruction filter bank systems and detailed analysis and FC-FB optimization methods are developed in [14]. These papers demonstrate the greatly increased flexibility and efficiency of FC-FB in communication signal processing, in comparison with the commonly used polyphase implementation structure.

The structure of FC-based flexible analysis filter bank (AFB) is illustrated in Fig. 1. We consider a case where the incoming high-rate, wideband signal is to be split into several narrowband signals with adjustable frequency responses and possibly also adjustable sampling rates. We are interested

in cases where the output signals are critically sampled or oversampled by a small factor. We also note that different subbands may be overlapping. The dual structure of Fig. 1 can be used for combining multiple low-rate, narrowband signals into a single wideband signal, following the frequency-division multiplexing principle.

Figure 1 includes sampling rate reduction by factors

$$R_k = N/L_k = N_S/L_{S,k} \quad (1)$$

where  $k$  is the subband index and the notation used for the lengths of the overlap-save processing blocks is indicated in Fig. 2. In other words, the sampling rate conversion factor is determined by the IFFT size, and can be configured for each subband individually. Naturally, the IFFT size determines the maximum number of frequency bins, i.e., the bandwidth of the subband. It is also possible to increase the subband output sampling rate by increasing the IFFT size by adding zero-valued bins outside the wanted subband frequency range.

Regardless of the specific processing applied for each subband, the decimation factor of the structure of Fig. 1 is given by (1) [9]. This means that the length of the FC output block is reduced by the same factor, and so is also the length of the overlapping part in the overlap-save processing. The input and output block lengths have to exactly match, taking into account the sampling rate conversion factor. It follows that in (1), both  $N/L_k$  and  $N_S/L_{S,k}$  have to be integers. For example, if  $N_S = 4N/5$ ,  $L_k$  has to be a multiple of 5 and the possible output sampling rates are multiples of  $5f_s/N$ , where  $f_s$  is the input sampling rate. So the configurability of the output sampling rate depends greatly on the choice of  $N$  and  $N_S$ .

There are two key parameters which have an effect on the spectral characteristics of the FC-FB scheme:

- The IFFT size  $L$  is defining how well the filter frequency response can be optimized. In general, increasing the value of  $L$  helps to improve the stopband attenuation.
- The overlap factor  $1 - L_S/L$ : In FC based multirate signal processing there is an inevitable cyclic distortion effect because the overlapping part of the processing block cannot be made big enough to absorb

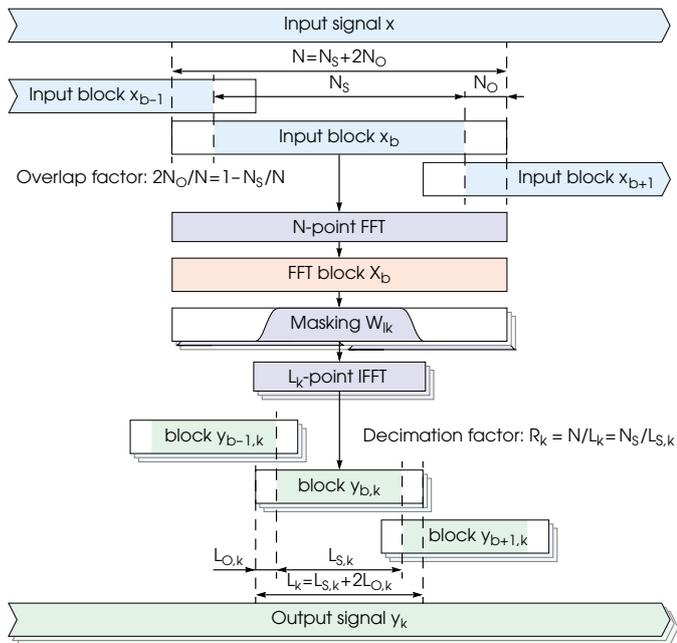


Fig. 2. Overlap-save processing.

the tails of the filter impulse response. Naturally, this effect can be reduced by increasing the overlap factor.

In [14] these effects were analyzed using a periodically time variant model for FC processing and effective tools for frequency response analysis and FC filter optimization were developed.

The FC-FB model can be used for generating and demodulating different kinds of communication waveforms. In our approach, the basic design is done for a filter channel with roll-off of 1. The FFT-domain weights consist of two symmetric transition bands and all stopband bins are set to zero. Figure 3(a) shows an FBMC/OQAM type multiplex of subchannels, which is constructed using such basic filters. The subchannel spacing is half of the overall bandwidth, which is equal to the IFFT length. Therefore, the subchannels are oversampled by two, which is also necessary for staggered, OQAM type subchannel processing. OQAM subcarrier signal model, in turn, is necessary for reaching (near) orthogonality of overlapping subcarriers in FB systems.

The transition band shape should be of the square-root Nyquist filter type. The root-raised-cosine (RRC) model can be used straightforwardly for constructing such transition bands. However, depending on the FC parameters, optimization of the weights may give significant improvement in the spectral characteristics [14].

One important feature of the FC-FB structure is that the transition band shape optimized for the basic case can be used for constructing filters with arbitrary bandwidths. In Figure 3(b) a filtered multi-tone (FMT) [15] type multiplex of non-overlapping subchannels is shown and Figure 3(c) shows a single-carrier transmission channel. In these cases, the normal QAM modulation can be used. The subchannel bandwidths and center frequencies can be independently tuned, with the resolution of the FFT bin spacing. Different types of multiplexes can be combined in a single FC-FB structure. It is also

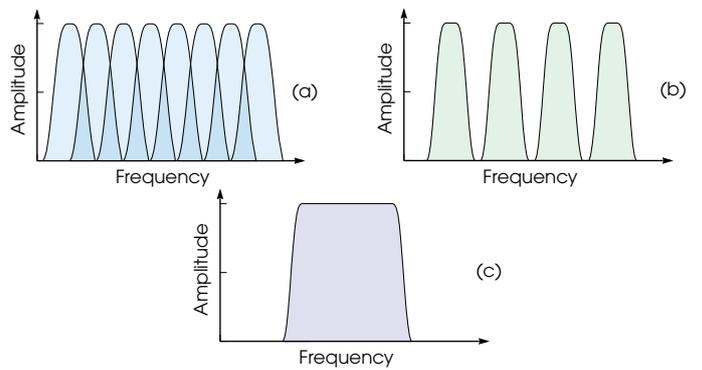


Fig. 3. Examples of different waveforms which can be implemented using the FC-FB structure. (a) FBMC/OQAM type multiplex of eight subchannel, (b) FMT type multiplex of four subchannels, (c) and single-carrier transmission channel.

possible to use different transition band shapes for different multiplexes, if so desired. The flexibility of the scheme can be used also for coarse frequency offset compensation by shifting the set of frequency bins connected to the subchannel processing functions. For time synchronization purposes, a linearly frequency-dependent phase response can be introduced to each subcarrier for timing offset correction.

### III. TIMING OFFSET COMPENSATION IN FC-FB

The proposed filter bank processing approach is particularly important for the base station transmitters and receivers, in which case there is a need to process simultaneously different FB-MC waveforms, and possibly also legacy signals. On the terminal side and in the ad-hoc context, a down-scaled version of the filter bank can be used, supporting a limited set of waveforms with reduced complexity.

Considering effective implementation of the base station receiver, it would be desirable to use a single FC-FB engine to process multiple uplink signals emerging from different non-synchronized mobiles. These signals may use different types of waveforms but they are assumed to be non-overlapping, i.e., a narrow guardband is inserted between different signal spectra. The relative frequency offsets are assumed to be small enough such that sufficient spectral isolation is maintained. As an example, with the 15 kHz subcarrier spacing of LTE, a few kHz frequency offsets can be tolerated without introducing significant cross-talk even with a single subcarrier guardband. Naturally, this depends on the filter bank design and expected power level differences between different uplink signals. Our focus here is on the relative timing offsets. The target is to compensate arbitrary fractional timing offsets in the range of  $[-T/2, T/2] = [-T_s/4, T_s/4]$ , where  $T$  is subchannel sample interval and  $T_s$  is the symbol interval.

In the FC-FB structure, the timing offset compensation can be implemented by adjusting the filtering delay through the phase response of each subchannel filter. A fractional timing offset (delay) of  $\tau_k$  in subchannel  $k$  can be compensated by introducing an additional linear phase response to the FFT-domain weights:

$$\hat{W}_{kl} = W_{kl} e^{j2\pi l \tau_k / L_k} \quad (2)$$

where  $W_{kl}$ ,  $l = -L_k/2, \dots, L_k/2 - 1$  are the original FFT-domain weights for subchannel  $k$ . The filter phase slope can be used for controlling the timing offset.

The basic FC-FB system can be designed for real FFT-domain weights, in which case the timing offset compensation would slightly increase the overall implementation complexity [14]. However, there are various reasons for considering complex weights, including (i) increased possibilities for the frequency response optimization and (ii) use of complex weights also used for subcarrier-wise channel equalization. All in all, the additional complexity due to the timing offset compensation in the FC-FB structure is relatively small.

#### IV. IMPLEMENTATION COMPLEXITY

The number of real multiplications for the traditional FBMC/OQAM scheme and FC-FB approach are given, respectively, by

$$C_{\text{FBMC}} = \frac{2C_{\text{FFT}} + 4KM + 4M_{\text{ac}}n}{M_{\text{ac}}} \quad \text{and} \quad (3a)$$

$$C_{\text{FC-FB}} = \frac{C_{\text{FFT}_{\text{long}}} + M_{\text{ac}}(C_{\text{FFT}_{\text{short}}} + 4[L - 1])}{M_{\text{ac}}L_S/2}, \quad (3b)$$

where  $M_{\text{ac}}$  is the number of active subcarriers,  $n$  is the number of coefficients of the complex equalizer, and  $C_{\text{FFT}}$  is the number of real multiplications required for the FFT/IFFT [14]. When the transform length is a power of two, the split-radix algorithm is considered to be the most efficient one, and the number of multiplications for a length  $N$  transform is expressible as

$$C_{\text{FFT}} = N(\log_2(N) - 3) + 4. \quad (4)$$

The number of real multiplications for several FFT algorithms for  $N$  not being a power of two is tabulated in [16].

#### V. NUMERICAL RESULTS

In this section we test the proposed method for timing offset control in FC-FB based FBMC/OQAM receiver using time-domain simulations. We choose a parametrization based on 5 MHz LTE with  $M = 512$  subcarriers, out of which 300 active subcarriers are used [2]. The 300 active subcarriers are scheduled in resource blocks of 12 subcarriers, i.e., there are 25 resource blocks (RBs). To accommodate non-synchronized usage of the subcarriers, for example, the highest subcarrier of each contiguous set of resource blocks allocated to a user should be left unused, as a guardband. FBMC/OQAM allows to transmit 15 multicarrier symbols in a subframe of two slots (1 ms), instead of 14 in basic LTE, which compensates the additional overhead in data transmission capacity due to the guardbands.

We consider three different parametrizations of the FC-FB system: (i)  $L = 10$ ,  $L_S = 7$ , (ii)  $L = 10$ ,  $L_S = 5$ , and (iii)  $L = 16$ ,  $L_S = 11$ . The subcarrier spacing is 15 kHz in all cases, so the FFT length becomes 2560 with  $L = 10$  and 4096 with  $L = 16$ . Case (i) uses a small IFFT length and relatively small overlap factor  $1 - L_S/L$ . In Case (ii) the overlap factor is increased to improve the spectral characteristics, with the cost of somewhat increased complexity. In Case (iii) the higher IFFT length gives more possibilities to optimize the spectral characteristics [14] with modest overlap factor. The drawback

of increased IFFT block length is increased sensitivity to fast fading. As a coarse metric for the implementation complexity, the number of real multiplications per symbol are 36, 50, and 37 for the three cases, respectively. These values are with complex FFT-domain weights, with real weights (without timing offset compensation) the corresponding values are 31, 43, and 31, respectively. For Cases (ii) and (iii), the weights of the FC-FB structure were optimized using the methods presented in [14]. With the parameters of Case (i), direct root-raised-cosine (RRC) weights are used since the optimization doesn't provide significant improvement; on the contrary the inband performance would be significantly degraded, as the optimization focuses on the out-of-band properties.

The disadvantage of the proposed way of timing offset compensation in the FC-FB structure is the inevitable cyclic distortion in the FC implementation of multirate filters. In optimized design the effect is symmetric within the processing block. With timing offset compensation, the cyclic truncation effect becomes more severe on one side of the effective linear impulse response [14]. This appears as reduced attenuation of out-of-band spectral components and increased inband interference. The latter effect is partly due to the nearly perfect-reconstruction nature of the filter bank and partly due to the cyclic distortion due to FC-FB implementation.

The inband and out-of-band effects can be seen in Fig. 4. The inband interference is just the mean-squared error in the subcarrier symbols, after timing offset compensation, as a function of the original timing offset. The out-of-band test situation includes just one active subcarrier, which is not synchronized to the receiver processing. This is basically modeling a narrowband legacy communication signal. Then we measure the interference power level at 2nd, 3rd, 5th, 10th, and 20th subcarrier (SC) from the active one, at the distances of 30 kHz, 45 kHz, 75 kHz, 150 kHz, and 300 kHz away from the active SC. The inband performance variations are rather small within the target range of timing offset control,  $\pm 0.25$  OQAM symbol intervals. Significant variations in the out-of-band performance can be seen, especially at higher SC distances, but the out-of-band interference attenuation is not severely compromised in any of the cases. We can also notice that in Case (i), with direct (non-optimized) RRC design, the out-of-band performance is reached with  $\pm 0.5$  symbol timing offset; the reason for this effect is explained in [14].

#### VI. CONCLUDING REMARKS

We have demonstrated that timing offset compensation by adjusting the FFT domain weights performs well in much wider range of timing offsets than what is actually needed for processing non-synchronized waveforms in an FC-FB engine. In comparison, with low-complexity subband processing after polyphase AFB, the demonstrated feasible timing offset compensation range is no more than  $\pm 10\%$  of the symbol interval [8]. With FC-FB, certain loss (typically well below 10 dB) in the suppression of out-of-band interferences is observed, but proper design of the FC-FB weights results in very good performance. Further improvement can be expected by modifying the FC-FB optimization algorithm to maximize the attenuation for worst-case timing offset, instead of the 0 timing offset case. Topics for future work include also frequency offset compensation methods as well as algorithms

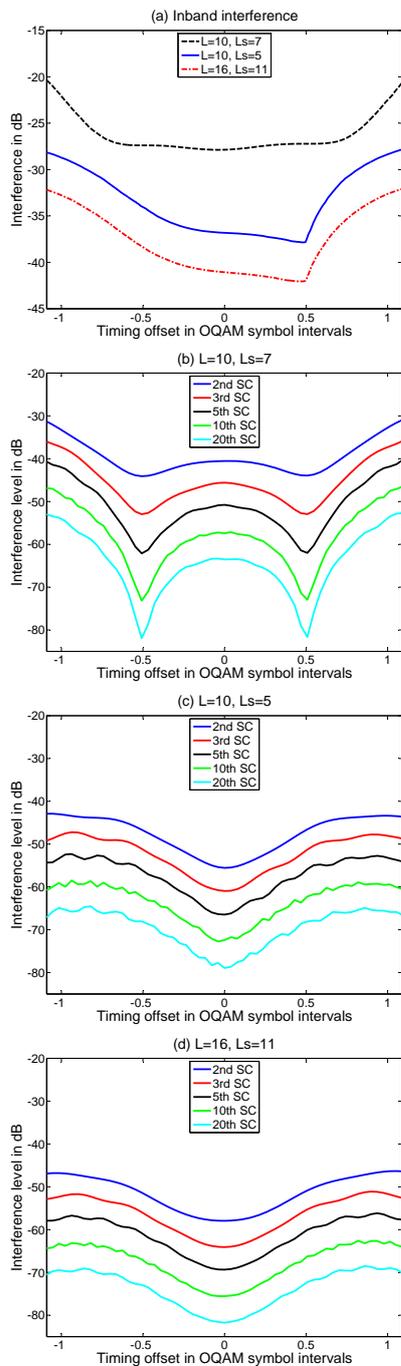


Fig. 4. Inband and out-of-band effects of timing offset control in the FC-FB implementation of FBMC/OQAM. (a) Inband effects. (b) Out-of-band effects in Case (i). (c) Out-of-band effects in Case (ii). (d) Out-of-band effects in Case (iii).

for estimating the frequency and timing offsets exploiting the FC-FB structure.

At system level, e.g., in cellular uplink transmission, the FC-FB based transmission modes allow fully asynchronous operation regarding timing offsets between transmissions and relatively coarse mutual frequency synchronization is sufficient. This allows the transmission of short data packets without initial synchronization, and maintaining the synchronization of idle stations is not critical. Also the latency due to

initial synchronization can be minimized. However, in case of continuous transmission in cell-based systems, there are good reasons for establishing synchronism between the up-link signals. These reasons include reduced signal processing complexity and minimization of the overheads in TDD and TDMA operation modes due to time-domain guard intervals between transmission bursts, which have to be able to absorb the timing uncertainties.

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