Design of 1-D Variable Fractional Delay All-pass Filters Using Stability Controlling

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Abstract—In this paper, a two-stage approach for the design of general 1-D stable variable fractional delay all-pass filters is proposed. The method takes the desired group delay range \([N-1, N]\), where \(N\) is the filter order. The design algorithm is decomposed into two design stages: Firstly, a set of fixed delay all-pass filters are designed by minimizing a set of objective functions defined in terms of the approximating error criterion and the filter stability constraint. Then, the design result is determined by fitting each of the fixed delay all-pass filter coefficients as 1-D polynomials. A design example together with its comparisons with those of the recent literatures is given to justify the effectiveness of the proposed design method.

I. INTRODUCTION

Since all-pass variable fractional delay (VFD) filters have unity magnitude response in the entire frequency band and achieve higher design accuracy than FIR VFD filters [1], the design and implementation of all-pass VFD filters have been receiving increasingly attention recently [3]-[12]. Because the stability problem must be taken into account to design the filters, so most of the design methods took the desired group delay range \([N-0.5, N+0.5]\) to guarantee the filter designed to be stable [2][3], where \(N\) is the filter order. However, this set will induce so big fractional delay error near the desired maximum group delay \(P\_N\), when the Least-Squares (LS) design is employed. Although the Weighted Least-Squares (WLS) design can improve the design accuracy [3][4], it is very difficult to find the satisfied weighting function. In [5], the desired group delay range is taken as \([N-1, N]\). The grievous problem in [5] is that when the desired group delay approximates to the minimum \(N-1\), the maximum modulus of filter poles will approximate to 1 then the filter stability will be destroyed.

In [11]-[12], we presented a simple two-stage approach for the design of general 1-D VFD IIR filters. The method can achieve the design of 1-D stable VFD all-pass filters. The design algorithm employs also the two-stage approach [11]-[15] to reduce the computational complexity. An illustrating design example shows that the method proposed can achieve better filter performances than the existing ones.

II. DESIGN PROBLEM

Let the frequency response of the desired 1-D VFD filter be

\[
H(d, \omega) = e^{-j\omega N} e^{j\omega d}, \quad \omega \in [0, \omega_c]
\]

where \(N\) is the maximum group delay, \(d\) denotes the fractional group delay that is continuously variable in the range \([-1, 0]\), and \(\omega_c<\pi\) is the filter cutoff frequency. Consider a 1-D stable and variable all-pass filter with coefficients \(a_i(p)\) \((i=1,2,\cdots,N)\), whose frequency response is expressed as

\[
H(d, \omega) = \frac{1 + \sum_{i=1}^{N} a_i(p)e^{j\omega i}}{1 + \sum_{i=1}^{N} a_i(p)e^{-j\omega i}} e^{-j\omega N} A(-j\omega, p) A(j\omega, p)
\]

Like that in [4], the variable coefficients \(a_i(p)\) \((i=1,2,\cdots,N)\) can be assumed as polynomial functions of \(p\) with the degree \(K\), i.e.

\[
a_i(p) = \sum_{k=0}^{K} x(i, k) p^k, \quad i=1,2,\cdots,N
\]

It is known that \(x(i,0)=0\) for all \(i=1,2,\cdots,N\).

The design task is to find the coefficients \(x(i,k)(i=1,2,\cdots,N, k=1,2,\cdots K)\) such that the corresponding \(H(d, \omega)\) approximates to \(H(d, \omega, p)\) and the filter stability condition is satisfied.

III. DESIGN METHOD

In this paper, two processes are employed in tandem to achieve the design of 1-D stable VFD all-pass filters.

A. First stage: Design a set of fixed fractional delay all-pass filters

Firstly, we sample uniformly \(M+1\) fractional delay points \(p_m=m/M-1 (m=0,1,\cdots,M)\) on \([-1, 0]\) to form \(M+1\) desired fixed fractional delay filters with respective frequency responses \(H_m(d, \omega, p_m)(m=0,1,\cdots,M)\) . From (1) and (2), we define \(M+1\) approximating error functions as

\[
e_i^2(p_m) = \int_0^{\omega_c} [A(-j\omega, p_m) - A(j\omega, p_m)e^{-j\omega N}]^2 d\omega, \quad m=0,1,\cdots,M
\]

It is clear that when \(e_i^2(p_m)\) reaches its minimum, the corresponding frequency response \(H(d, \omega, p_m)\) approximates to the desired one \(H(d, \omega, p_m)\) . Furthermore, like those presented in [11]-[12], we define \(M+1\) quadratic functions for...
the stability constraint as
\[ e^2(p_m) = \sum_{m=1}^{N} e^2(p_m), \quad m = 0,1,\cdots M \] (5)

From [11][12], it is known that if \( e^2(p_m) \) is less than 0.5, the sufficient filter stability constraint condition [16]
\[ R_x(A(j\omega, p_m)) > 0, \quad \omega \in [0,\pi) \] (6)
is satisfied. By combining (4) and (5), we define \( M+1 \) objective functions as
\[ e^2(p_m) = e^2(p_m) + W_s e^2(p_m), \quad m = 0,1,\cdots M \] (7)
where \( W_s \) is an adjustable positive number and is called stability control parameter (SCP) [11][12]. Therefore, the task of designing \( M+1 \) stable fixed and all-pass filters is to minimize the objective functions \( e^2(p_m) \) for \( m = 0,1,\cdots M \). It is observed that to facilitate the design of stable filters, \( W_s \) must be taken big enough. On the other hand, from (7), an oversize \( W_s \) may induce a big approximating error \( e^2(p_m) \). Therefore, to harmonize the both stability and accuracy, \( W_s \), must be taken to be suitable.

From (8) and (10), the major computation involves calculating the inverse matrices \( [T(p_m) + 0.5W_sI]^{-1} \) for \( m = 0,1,\cdots M \) and \( C^{-1} \), with matrix dimensions respectively \( N \times N \) and \( K \times K \). Since a \( n \times n \) inverse matrix can be obtained using \( n^3 \) floating-point operations (flops), the computational complexity of the proposed method is approximately \( (M+1)N^3 + K^3 \) flops.

IV. DESIGN EXAMPLE

In this section, an illustrating design example simulated using MATLAB on a PC is given. The design specifications are chosen the same as those adopted in [4]: \( \omega_s = 0.9\pi, N=35 \), and \( K=5 \). To evaluate the performance, the variable frequency response error, variable fractional delay error, maximum frequency response error (MFRE), normalized root mean square error (RMSE), and maximum fractional delay error (MFDE) are defined also the same as those adopted in [4].

The sampling scale \( M \) is chosen 25. From Section III, \( W_s \) must be taken to be suitable. Fig.1(a) shows the maximum pole modulus (MPM) for \( p \in [-1,0] \) of the obtained filter when \( W_s \) is taken 0. It is known that when \( p = -1 \), MPM equal 1 then the filter is unstable. It is found in our design process that if \( W_s \) is larger than about \( 5.0 \times 10^{-12} \) the filter obtained will be stable and different \( W_s \), will yield different filter performances. Fig.2 shows that MFRE changes and the maximum of MPM for all \( p \in [-1,0] \) decreases as \( W_s \), increases from \( 5.0 \times 10^{-12} \) to \( 5.0 \times 10^{-10} \). It can be seen that MFRE will reach its minimum if \( W_s = 1.4 \times 10^{-10} \).

Fig.1(b) gives also MPM for \( p \in [-1,0] \) when \( W_s = 1.4 \times 10^{-10} \). The maximum is 0.9946, hence the filter is stable. The variable frequency response error and variable fractional delay error at the frequency range \([0,0.9\pi]\) are shown respectively in Fig.3 and Fig.4. Under the same computing condition as: 201 discrete normalized frequency

895
points on $\omega \in [0, 0.9\pi]$ and 61 fractional normalized delay points on $p \in [-1, 0]$, the filter performance indexes MFRE, RMSE, and MFDE comparing with those published in [4] are listed in Table I. It is known that the method proposed obtains the 1-D VFD all-pass filter with better performances than the both LS and WLS designs of [3][4], while the same design specifications are taken.

In addition, while $W_s$ is taken as $W_s = 1.4 \times 10^{-10}$, the proposed approach needs only less than 1 second CPU time on a PC (Pentium IV, 1.8GHz) to design the above filter. In practical design, from Fig.2, it must repeat the design algorithms many times to find the optimum $W_s$ such that the MFRE reaches its minimum. However, provided design experiences are accumulated and some skills are adopted, the time to find the optimum $W_s$ may be decreased.

Fig. 3. Variable frequency response error (dB).

V. CONCLUSION

In this paper, a stability controlling based two-stage approach for the design of 1-D stable VFD all-pass filters has been proposed. Being differ from most of the design method using the desired group delay range $[N-0.5, N+0.5]$, the range $[N-1, N]$ [5] is taken in this paper, where $N$ is the filter order. The proposed method employs the stability control parameter (SCP) proposed in [11][12] to guarantee the designed filter to be stable. The design algorithm is performed by solving in tandem two linear matrix equations and has low computational complexity. An illustrating design example shows that the optimum SCP such that the maximum frequency response error (MFRE) reaches its minimum can be found by repeating the design algorithms, and the filter obtained finally has better performances than those reported in the recent literatures [3][4].
REFERENCES


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TABLE I. DESIGN RESULT COMPARED WITH THOSE OF [3][4]

<table>
<thead>
<tr>
<th>Method</th>
<th>MFRE (dB)</th>
<th>RMSE (%)</th>
<th>MFDE</th>
<th>Filter coefficients</th>
<th>Desired group delay range</th>
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</thead>
<tbody>
<tr>
<td>L.S design of [3]</td>
<td>-65.14</td>
<td>0.001146</td>
<td>0.0369</td>
<td>210</td>
<td>[34.5, 35.5]</td>
</tr>
<tr>
<td>WLS design of [3]</td>
<td>-67.49</td>
<td>0.000996</td>
<td>0.0291</td>
<td>210</td>
<td>[34.5, 35.5]</td>
</tr>
<tr>
<td>L.S design of [4]</td>
<td>-74.95</td>
<td>0.000562</td>
<td>0.0158</td>
<td>175</td>
<td>[34.5, 35.5]</td>
</tr>
<tr>
<td>WLS design of [4]</td>
<td>-86.43</td>
<td>0.000654</td>
<td>0.0067</td>
<td>175</td>
<td>[34.5, 35.5]</td>
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<tr>
<td>Proposed design</td>
<td>-89.93</td>
<td>0.000497</td>
<td>0.0022</td>
<td>175</td>
<td>[34.0, 35.0]</td>
</tr>
</tbody>
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