An Efficient and Simple Method for Designing Prototype Filters for Cosine-Modulated Pseudo-QMF Banks

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***Abstract*—We present a new method to design prototype filters for conventional cosine-modulated pseudo-quadrature mirror filter (QMF) banks. This method is based on windowing, and sets the 3-dB cutoff frequency of the filter obtained at $\pi/2M$. In this way, the filter bank performance can be significantly improved compared to other existing design methods.**

***Index Terms*—Channel bank filters, filtering theory.**

I. Introduction

THE DESIGN of M-channel maximally decimated quadrature mirror filter (QMF) banks is a topic of multirate digital signal processing that has received widespread attention (as examples, see references included in [1] or [2]). In cosine-modulated filter banks, analysis and synthesis filters are cosine-modulated versions of a lowpass prototype filter. Thus, it is one of the most effective techniques for constructing both the analysis and synthesis filter banks because the design of the whole filter bank reduces to that of the prototype filter.

Several efficient methods have been proposed to facilitate the design of this prototype filter. In the method proposed by Creusere and Mitra [3], the prototype filter length, relative error weighting, and stopband edge are fixed before the optimization procedure is started, while the passband edge is adjusted to minimize an objective function. In the Kaiser Window Approach (KWA) to the design of prototype filters of cosine-modulated filter banks [4], the design process of the prototype filter is reduced to the optimization of the cutoff frequency in Kaiser window designs.

In this letter, a new prototype filter design technique for conventional cosine-modulated pseudo-QMF banks [1], [5] based on windowing is proposed. This technique controls the 3-dB point of the prototype magnitude response and sets it approximately at $\pi/2M$. In this way, and as the results obtained show, it is possible to reduce amplitude distortion and aliasing errors introduced in the bank by using prototype filters with the same order than that obtained with other existing design techniques. An example is given to show that the prototype filters obtained perform successfully.

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II. Conditions for the Nearly Perfect Reconstruction Property

The conditions for approximate reconstruction in a filter bank can be stated in terms of the linear-phase prototype filter $H(e^{j\omega})$ and the overall distortion transfer function $T(e^{j\omega})$ as follows [1], [5]:

$$\left|H\left(e^{j\omega}\right)\right| \approx 0 \qquad |\omega| > \pi/M, \tag{1}$$

$$\left|T\left(e^{j\omega}\right)\right| = \left|\sum_{k=0}^{M-1} F_k\left(e^{j\omega}\right)\cdot H_k\left(e^{j\omega}\right)\right| \approx 1 \tag{2}$$

where $H_k(e^{j\omega})$ and $F_k(e^{j\omega})$ are cosine-modulated versions of $H(e^{j\omega})$. The cosine-modulation scheme proposed in [1], [5] guarantees that if $G(z) = H^2(z)$ is approximately a $2M$th-band linear-phase FIR filter [1], [6], the amplitude distortion becomes very small. In this way, the magnitude response of the prototype filter must achieve approximately the value $1/\sqrt{2}$ at $\omega = \omega_{c,\,3\,\mathrm{dB}} = \pi/2M$, i.e. [7]

$$\left|H\left(e^{j\pi/(2M)}\right)\right| \approx 1/\sqrt{2}. \tag{3}$$

III. Prototype Filter Design Based on Windowing

Let $h[n]$ be the linear-phase prototype FIR filter of odd-length $(N+1)$ obtained by windowing $h[n] = h_i[n]\cdot w[n]$, where $w[n]$ is the $(N+1)$-length window and

$$h_i[n] = \frac{\sin((n-N/2)\cdot\omega_{c,\,6\,\mathrm{dB}})}{\pi\cdot(n-N/2)}, \qquad n \in Z \tag{4}$$

is the shifted impulse response of an ideal lowpass filter. Both sequences $h_i[n]$ and $w[n]$ are symmetric with respect to $n = N/2+1$, and the obtained filter $h[n]$ satisfies condition that the magnitude response of $H(e^{j\omega})$ achieves the value 1/2 at $\omega = \omega_{c,\,6\,\mathrm{dB}}$. This angular frequency $\omega_{c,\,6\,\mathrm{dB}}$ is an initial parameter in equation (4), and it must be chosen so that amplitude distortion is reduced. By other words, the prototype filter must be designed satisfying equation (3) and the value of $\omega_{c,\,3\,\mathrm{dB}}$ must be located approximately at $\pi/2M$.

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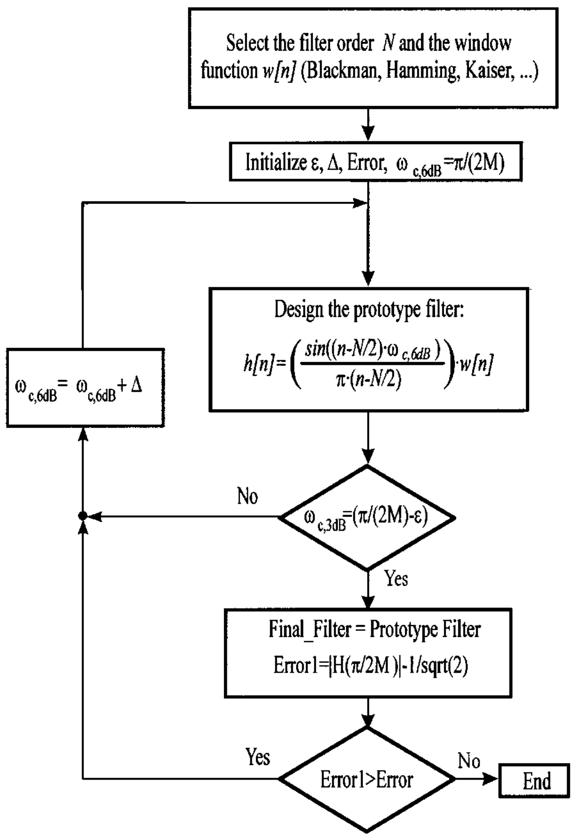

Fig. 1. Flowchart of the proposed algorithm.

Based on these facts, we propose a new technique to design prototype filters that modifies the cutoff frequency $\omega_{c,\,6\,\mathrm{dB}}$ of the ideal lowpass filter $h_i[n]$ in order to obtain the 3-dB cutoff frequency ($\omega_{c,\,3\,\mathrm{dB}}$) of $h[n]$ located approximately at $\pi/2M$. The statement of the problem can be raised as follows: given N and the finite-length window $w[n]$, adjust the parameter $\omega_{c,\,6\,\mathrm{dB}}$ to find the best $h[n]$ that yields the smallest

$$\phi = \left| \left| H\left(e^{j\pi/2M}\right)\right| - 1/\sqrt{2}\right|. \tag{5}$$

In this way, the linear-phase FIR filter designed by windowing satisfies approximately the power complementary property, i.e.,

$$\begin{cases} \left|H\left(e^{j\omega}\right)\right|^2 + \left|H\left(e^{j(\pi/M-\omega)}\right)\right|^2 \approx 1, & 0 \le \omega \le \pi/M \\ \left|H\left(e^{j\omega}\right)\right|^2 + \left|H\left(e^{j(-\pi/M-\omega)}\right)\right|^2 \approx 1, & -\pi/M \le \omega \le 0. \end{cases} \tag{6}$$

The proposed algorithm has been simulated in MATLAB, and our experiments have shown that rather good results can be obtained. A flowchart of the complete routine is shown in Fig. 1.

IV. Design Example

We use different techniques to design 439-length prototype filters for 32-channel cosine-modulated filter banks. The specifications are similar to that of the MPEG audio coder [8, p. 397], i.e., the stopband attenuation A_S of the prototype filters is around 100 dB. This example has been chosen because it acts as reference to compare our technique with those developed in [3], [4], and [8]. First, a prototype filter has been obtained using the proposed technique with a Kaiser-window ($\beta = 10.061\,26$). This filter satisfies that $\omega_{c,\,3\,\mathrm{dB}} \approx \pi/(2M)$. Fig. 2 shows the magnitude response of this filter. The resulting function $|T(e^{j\omega})|$ is approximately flat (Fig. 3) with peak amplitude distortion $R_{pp} = 3.385 \cdot 10^{-3}$. The maximum aliasing error [1] is very small: $E_a = 2.611 \cdot 10^{-7}$. Secondly, a prototype filter is designed using the Creusere and Mitra technique. The obtained results are the following: $R_{pp} = 5.299 \cdot 10^{-3}$ and $E_a = 2.586 \cdot 10^{-6}$. Finally, we use the KWA technique to design the third prototype filter. The resulting filter bank shows a peak amplitude distortion $R_{pp} = 3.443 \cdot 10^{-3}$ and a maximum aliasing error $E_a = 2.616 \cdot 10^{-7}$. From these results, it can be seen that fixed both the stopband attenuation and the order of the prototype filter, we have obtained with the proposed technique amplitude distortion and aliasing errors which are similar or even better than those obtained with the techniques proposed in [3] and [4]. Note that the optimization procedure proposed in this letter is also simple conceptually and computationally.

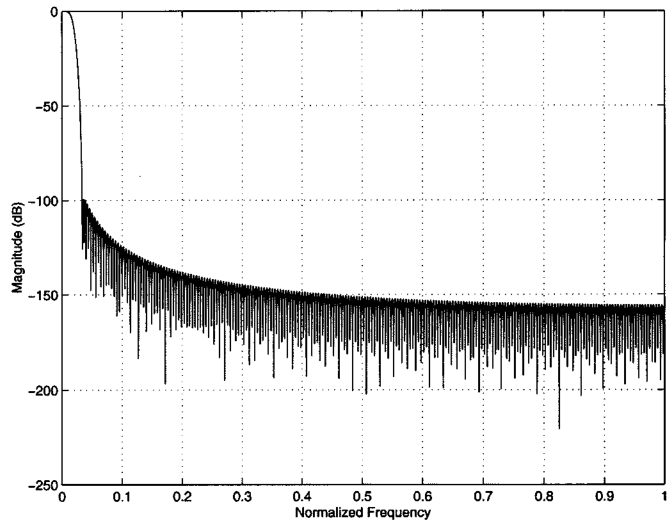

Fig. 2. Magnitude response for the prototype filter $P(z)$.

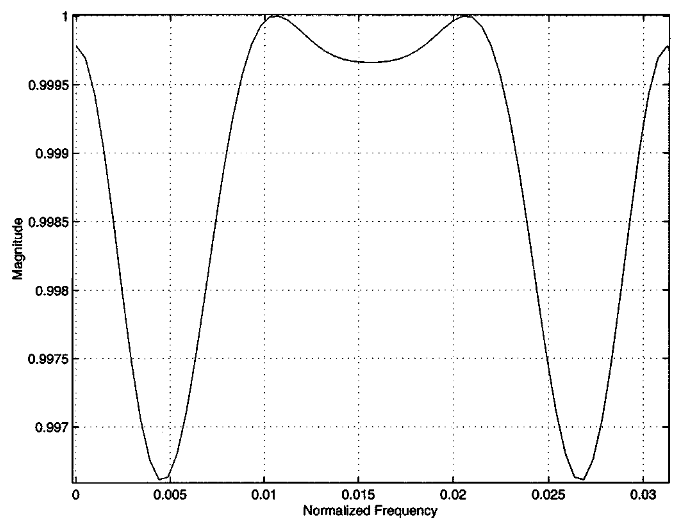

Fig. 3. Magnitude response plot of $T(e^{j\omega})$ (periodic $\pi/32$) in the interval $[0,\ \pi/32]$.

V. Conclusions

We have presented a simple new method based on the windowing technique for designing prototype filters for cosine-modulated pseudo-QMF banks. Our method varies the value of the 6-dB cutoff frequency of the ideal filter so that the final filter designed has its 3-dB cutoff frequency located approximately at $\pi/2M$. The results obtained and shown as examples for MPEG audio coders, confirm the validity of the proposed technique.

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